AP Calculus AB
Chapter 4, Section 4

The Fundamental Theorem of Calculus
2013 - 2014

The Fundamental Theorem of Calculus

- Discovered independently by Isaac Newton and Gottfried Leibniz

- Basically says that derivatives and integrals are inverses of each other, much like multiplication and division.

- While we find the derivative of a function to be defined as $\frac{\Delta y}{\Delta x}$, we inversely say the area under the curve (since we use rectangles) is $\Delta y \Delta x$.

The Fundamental Theorem of Calculus

- If a function *f* is continuous on the closed interval [a, b] and F is an antiderivative of *f* on the interval [a, b], then

$$\int_a^b f(x)dx = F(b) - F(a)$$

- It is not necessary to include the addition of "c" when finding the antiderivative since they will just cancel out when you subtract the functions.

Evaluate the definite integral

$$\int_{1}^{2} (x^2 - 3)dx$$

Evaluate the definite integral

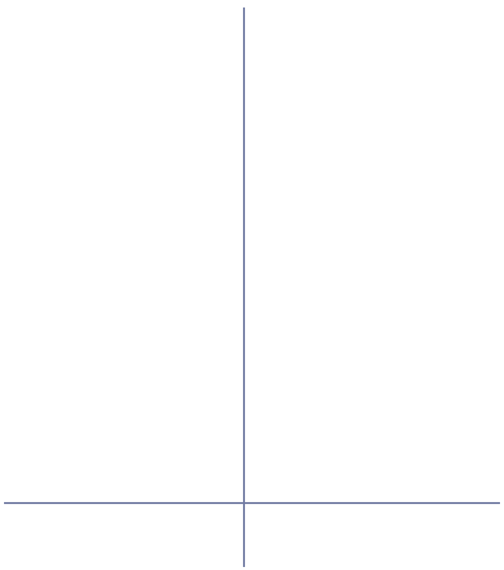
$$\int_{1}^{4} 3\sqrt{x}\,dx$$

Evaluate the definite integral

$$\int_0^{\pi/4} \sec^2 x dx$$

A definite integral involving Absolute Value

- Evaluate $\int_0^2 |2x - 1|\, dx$



Using the Fundamental Theorem to Find Area

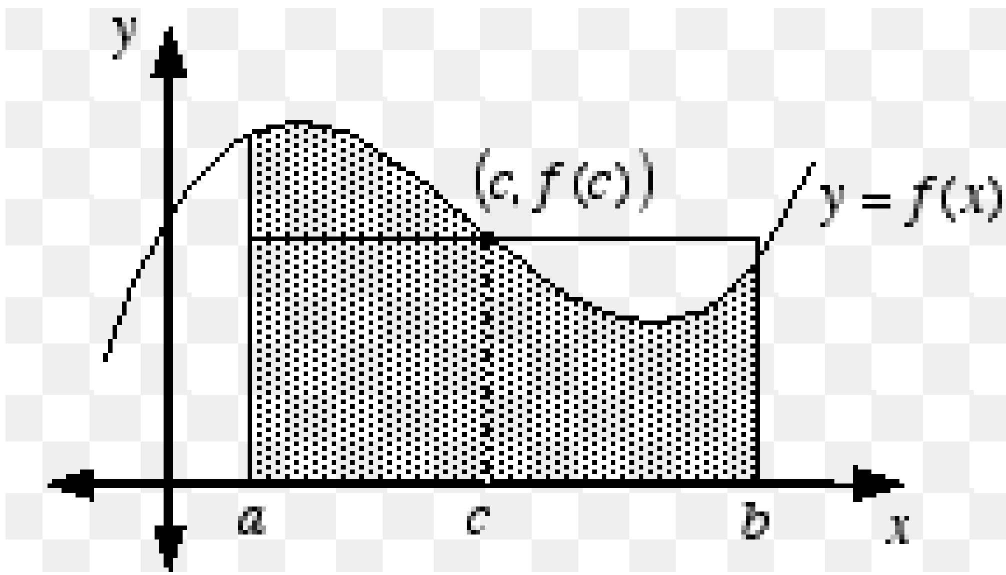
- Find the area of the region bounded by the graph of $y = 2x^2 - 3x + 2$, the x-axis, and the vertical lines x=0 and x=2.

Finding the Area of a Region

- Find the area of the region bounded by the graphs of the following equations: $y = x^3 + x, \;\; x = 2, \;\; y = 0$

Mean Value Theorem for Integrals

- The mean value theorem for integrals implies that somewhere in the interval of the integral, there is a point that if you made a rectangle that includes that point, the area of that rectangle would be the area under the curve.



Mean Value Theorem for Integrals

- If *f* is continuous on the closed interval [a, b], then there exists a number *c* in the closed interval [a, b] such that

$$\int_a^b f(x)dx = f(c)(b-a)$$

- Can be referred to as the MVTI

Using Mean Value Theorem for Integrals

- Find the value(s) of *c* guaranteed by the Mean Value Theorem for Integrals for the function over the given interval.

$$f(x) = \frac{x^2}{4}, \qquad [0, 6]$$

Average Value of a Function

- The value *f(c)* given in the MVT for Integrals is called the **average value** of f on the interval [a, b]

- If *f* is integrable on the closed interval [a, b], then the **average value** of *f* on the interval is

$$\frac{1}{b-a}\int_a^b f(x)dx$$

Finding the Average Value of a Function

- Find the average value of $f(x) = 3x^2 - 2x$ on the interval $[1, 4]$.

The Second Fundamental Theorem of Calculus

- If $f(x)$ is continuous on $[a, b]$, then the derivative of the function $F(x) = \int_a^x f(t)dt$ is:

$$\frac{dF}{dt} = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

Khan Academy Video

- Integral Calculus
- https://www.khanacademy.org/math/integral-calculus/indefinite-definite-integrals/fundamental-theorem-of-calculus/v/fundamental-theorem-of-calculus

Second Fundamental Theorem of Calculus: Chain Rule

- Sometimes the chain rule has to be applied to the second fundamental theorem of Calculus:

$$\frac{d}{dx}\left[\int_{a}^{u(x)} f(t)dt\right] = f[u(x)] \cdot u'(x)$$

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Practice

- Find the derivative of $F(x) = \int_{\pi/2}^{x^3} \cos t \; dt$

Apply Second Fundamental Theorem

- If $F(x) = \int_0^{x^2} \sqrt{t+3}\,dt$, what is $F'(x)$?

A. $\sqrt{x^2+3}$

B. $\frac{1}{2\sqrt{x^2+3}}$

C. $2x\left(\sqrt{x^2+3}\right)$

D. $\frac{2\left(x^2+3\right)^{3/2}}{3}$

E. *None of the above*

Ch 4.4 Homework

- Pg. 291 – 292, #'s: 1 – 45 every other odd, 47, 51, 63