# More About Quadrilaterals

ACTIVITY 16

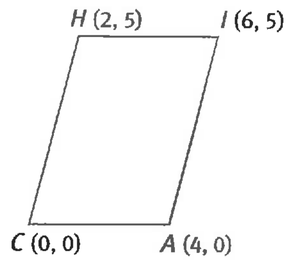
## A 4-gon Conclusion
### Lesson 16-1 Proving a Quadrilateral Is a Parallelogram

**Learning Targets:**
- Develop criteria for showing that a quadrilateral is a parallelogram.
- Prove that a quadrilateral is a parallelogram.

**SUGGESTED LEARNING STRATEGIES:** Think-Pair-Share, Group Presentation, Discussion Groups, Visualization

In a previous activity, the definition of a parallelogram was used to verify that a quadrilateral is a parallelogram by showing that both pairs of opposite sides are parallel.

**1.** Given quadrilateral *CHIA*:




**a.** Find the slope of each side.

The slope of $\overline{HI}$ = the slope of $\overline{CA} = 0$;
The slope of $\overline{CH}$ = the slope of $\overline{IA} = \frac{5}{2}$

**b.** Use the slopes to explain how you know quadrilateral *CHIA* is a parallelogram.

If two lines have the same slope, then they are parallel. A parallelogram is a quadrilateral with both of opposite sides parallel.

My Notes

**MATH TIP**

**Slope Formula**

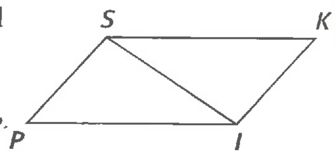
Given $A(x_1, y_1)$ and $B(x_2, y_2)$

Slope of $\overline{AB}$: $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

**MATH TIP**

Once a theorem has been proven, it can be used to justify other steps or statements in proofs.

**2.** Given quadrilateral *SKIP* with $SK = IP$ and $KI = SP$.




**a.** $\triangle PSI \cong$ $\triangle KIS$. Explain.

SI=IS by the Reflexive Prop.
triangles congruent by SSS

**b.** $\angle SIP \cong$ $\angle KSI$ and $\angle PSI \cong$ $\angle KIS$. Explain.

by CPCTC

**c.** $\overline{SK} \parallel \overline{IP}$ and $\overline{KI} \parallel \overline{SP}$ because ______________.

Alternate interior angles are congruent.

**d.** Complete the theorem.

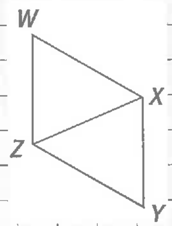
**Theorem** If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a __________.

parallelogram.

**3.** Given quadrilateral *WALK* with coordinates $W(8, 7)$, $A(11, 3)$, $L(4, 1)$, and $K(1, 5)$. Use the theorem in Item 2 to show that ▱*WALK* is a parallelogram.

$WA = 5$, $LK = 5$, $AL = \sqrt{53}$, $WK = \sqrt{53}$
Both pairs of opposite sides are congruent.

My Notes




4. Given $\square WXYZ$ with $\overline{WX} \parallel \overline{ZY}$ and $\overline{WX} \cong \overline{ZY}$.

   a. $\triangle WZX \cong$ ~~$\triangle YXZ$~~ Explain.

   by SAS

   b. **Construct viable arguments.** Explain why $\overline{WZ} \parallel \overline{XY}$.

   $\angle WZX \cong \angle YXZ$ by CPCTC. $\angle WZX$ and $\angle YXZ$ are alternate interior angles.

   c. Complete the theorem.

   **Theorem** If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a ________.

   parallelogram

5. Given $\square GOLD$ with coordinates $G(-1, 0)$, $O(5, 4)$, $L(9, 2)$, and $D(3, -2)$. Use the theorem in Item 4 to show that $\square GOLD$ is a parallelogram.

   $GO = LD = \sqrt{52}$; the slope of $\overline{GO}$ and $\overline{LD}$ is $\frac{2}{3}$

   $OL = GD = \sqrt{20}$; the slope of $\overline{OL}$ and $\overline{GD}$ is $\frac{1}{2}$

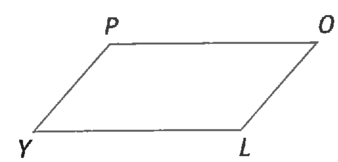
Now you can prove a theorem that can be used to show that a given quadrilateral is a parallelogram.

## Example A

**Theorem** *If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.*

**Given:** $\square POLY$ with $\angle P \cong \angle L$ and $\angle O \cong \angle Y$

**Prove:** $\square POLY$ is a parallelogram.




| Statements | Reasons |
|---|---|
| 1. $\square POLY$ with $\angle P \cong \angle L$ and $\angle O \cong \angle Y$ | 1. Given |
| 2. $m\angle P = m\angle L$ and $m\angle O = m\angle Y$ | 2. Def. of congruent angles |
| 3. $m\angle P + m\angle O + m\angle L + m\angle Y = 360°$ | 3. The sum of the measures of the interior angles of a quadrilateral is 360°. |
| 4. $m\angle P + m\angle O + m\angle P + m\angle O = 360°$ | 4. Substitution Property |
| 5. $2m\angle P + 2m\angle O = 360°$ | 5. Simplify. |
| 6. $m\angle P + m\angle O = 180°$ | 6. Division Property of Equality |
| 7. $m\angle P + m\angle Y + m\angle P + m\angle Y = 360°$ | 7. Substitution Property |
| 8. $2m\angle P + 2m\angle Y = 360°$ | 8. Simplify. |

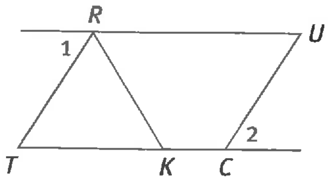
| Statements | Reasons |
|---|---|
| 9. $m\angle P + m\angle Y = 180°$ | 9. Division Property of Equality |
| 10. $\overline{PY} \parallel \overline{OL}$ and $\overline{PO} \parallel \overline{YL}$ | 10. If two lines are intersected by a transversal and a pair of consecutive interior angles are supplementary, then the lines are parallel. |
| 11. $\square POLY$ is a parallelogram. | 11. Def. of a parallelogram |

## Try These A

Write a proof using the theorem in Example 1 as the last reason.

**Given:** $\overline{RT} \cong \overline{RK}$
$\angle RKT \cong \angle U$
$\angle 1 \cong \angle 2$

**Prove:** $\square TRUC$

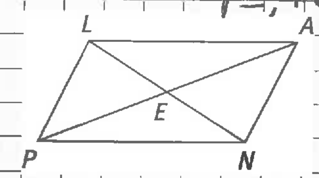



My Notes

Try These A

| Statements | Reasons |
|---|---|
| $\overline{RT} \cong \overline{RK}$ | Given |
| $\angle RKT \cong \angle T$ | Isosceles $\triangle$ Theorem |
| $\angle RKT \cong \angle U$ | Given |
| $\angle T \cong \angle U$ | Transitive Property |
| $\angle 1 \cong \angle 2$ | Given |
| $\angle TRU \cong \angle UCT$ | Supplements of congruent $\angle$s are congruent. |
| $\square TRUC$ | Both pairs of opp $\angle$s are $\cong$, the quad. is a para. |

6. Given $\square PLAN$ whose diagonals, $\overline{PA}$ and $\overline{LN}$, bisect each other.




Complete the statements.

a. $\triangle LEP \cong$ $\triangle NEA$ and $\triangle LEA \cong$ $\triangle NEP$. Explain.

Use the definition of bisect and the Vertical Angle Theorem to prove triangles congruent by SAS.

b. $\angle ALE \cong$ $\angle PNE$ and $\angle ELP \cong$ $\angle ENA$. Explain.

CPCTC

c. Explain how the information in part b can be used to prove that $\square PLAN$ is a parallelogram.

Congruent alt. interior angles implies that there are two pairs of parallel opposite sides, which is the definition of a parallelogram.

d. Complete the theorem.

**Theorem** If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

7. Given $\square THIN$ with coordinates $T(3, 3)$, $H(5, 9)$, $I(6, 5)$, and $N(4, -1)$.

a. Find the coordinates for the midpoint of each diagonal.

The coordinates of the midpoint of $\overline{TI}$ and $\overline{HN}$ are both (4.5, 4)

b. Do the diagonals bisect each other? Explain.

Yes. The diagonals have the same midpoint. A segment is bisected by its midpoint.

c. The best name for this quadrilateral is:

A. quadrilateral B. kite C. trapezoid D. parallelogram

My Notes

**8.** Summarize this part of the activity by making a list of the five ways to prove that a quadrilateral is a parallelogram.

- Both pairs of opposite sides are parallel.
- Both pairs of opposite sides are congruent.
- One pair of opposite sides congruent and parallel.
- Both pairs of opposite angles congruent.
- The diagonals bisect each other.

## Check Your Understanding

**9.** Explain why showing that only one pair of opposite sides of a quadrilateral are parallel is not sufficient for proving it is a parallelogram.

**10.** Three of the interior angle measures of a quadrilateral are 48°, 130°, and 48°. Is the quadrilateral a parallelogram? Explain.

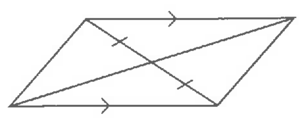
9) A trapezoid has one pair of parallel sides, but it is not a parallelogram.

10) $360° - 2(48) - 130° = 134°$; The fourth angle would have to equal 130°. It is not a quadrilateral

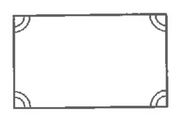
## LESSON 16-1 PRACTICE

**Make use of structure.** Tell what theorem can be used to prove the quadrilateral is a parallelogram. If there is not enough information to prove it is a parallelogram, write "not enough information."

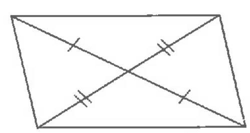
**11.**



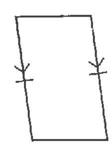
**12.**



**13.**



**14.**



11) not enough information

12) If both pairs of opposite angles of a quadrilateral are congruent, then the quad is a parallelogram.

13) If the diagonals of a quad bisect each other, then the quad is a parallelogram.

14) If one pair of opposite sides of a quad are parallel and congruent, then the quad is a parallelogram.

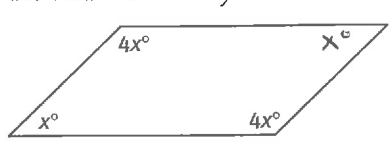
Three vertices of a parallelogram are given. Find the coordinates of the fourth vertex.

**15.** (1, 5), (3, 3), (8, 3) (6,5)

**16.** (−5, 0), (−2, −4), (3, 0) (6,−4)

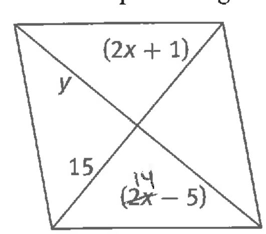
Find the values of $x$ and $y$ that make the quadrilateral a parallelogram.

**17.**




$4x + 4x + x + x = 360$
$10x = 360$
$\boxed{x = 36}$

**18.**




$2x + 1 = 15$
$\boxed{x = 7}$
$y = 2x - 5$
$= 2(7) - 5$
$\boxed{y = 9}$

My Notes

## Learning Targets:

- Develop criteria for showing that a quadrilateral is a rectangle.
- Prove that a quadrilateral is a rectangle.

**SUGGESTED LEARNING STRATEGIES:** Think-Pair-Share, Create Representations, Group Presentation, Discussion Groups

**1.** Complete the following definition.

A rectangle is a parallelogram with ________.
four right angles

**2. a.** Complete the theorem.

**Theorem** If a parallelogram has one right angle, then it has four right angles, and it is a rectangle

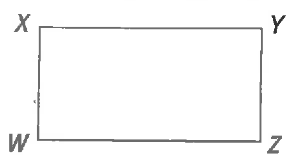
**b.** Use one or more properties of a parallelogram and the definition of a rectangle to explain why the theorem in Item 1 is true.
Since consecutive angles are supplementary in a parallelogram, the two angles consecutive to a given right ∠ will be right ∠s. Since opposite ∠s in a parallelogram are congruent, the angle opposite the right angle is a right angle. By definition, a quadrilateral with four right angles is a rectangle.

**3.** Given ▱*WXYZ*.

**a.** If ▱*WXYZ* is equiangular, then find the measure of each angle.
90°

**b.** Complete the theorem.

**Theorem** If a quadrilateral is equiangular, then it is a rectangle




**4. Make sense of problems.** Identify the hypothesis and the conclusion of the theorem in Item 3. Use the figure in Item 3.

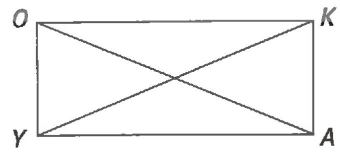
**Hypothesis:** ▱WXYZ is equiangular
**Conclusion:** ▱WXYZ is a rectangle

**5.** Write a proof for the theorem in Item 3.

| Statements | Reasons |
|---|---|
| ▱WXYZ is equiangular | Given |
| $m\angle W = m\angle X = m\angle Y = m\angle Z$ | Def. of equiangular |
| $m\angle W + m\angle X + m\angle Y + m\angle Z = 360°$ | The four ∠s of a quadrilateral add to 360° |
| $4m\angle W = 360°$ | Substitution Property |
| $m\angle W = 90°$ | Division Property |
| ▱WXYZ | If a parallelogram has one right angle, it is a rectangle |

My Notes

6. Given ▱$OKAY$ with congruent diagonals, $\overline{OA}$ and $\overline{KY}$.




a. List the three triangles that are congruent to $\triangle OYA$, and the reason for the congruence.

$\triangle OYA \cong \triangle KAY \cong \triangle YOK \cong \triangle AKO$ by SSS

b. List the three angles that are corresponding parts of congruent triangles and congruent to $\angle OYA$.

$\angle OYA \cong \angle KAY \cong \angle YOK \cong \angle AKO$

c. Find the measure of each of the angles in part b.

$m\angle OYA = m\angle KAY = m\angle YOK = m\angle AKO = 90°$

d. Complete the theorem.

**Theorem** If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle

7. Given ▱$ABCD$ with coordinates $A(1, 0)$, $B(0, 3)$, $C(6, 5)$, and $D(7, 2)$.
a. Show that ▱$ABCD$ is a parallelogram.
b. Use the theorem in Item 6 to show that ▱$ABCD$ is a rectangle.

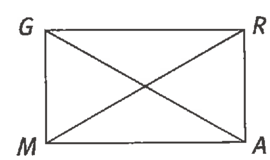
$AC = BD = \sqrt{50}$; the diagonals are congruent

8. Write a two-column proof using the theorem in Item 6 as the last reason.

**Given:** ▱$GRAM$

$\triangle GRM \cong \triangle RGA$

**Prove:** ▱$GRAM$ is a rectangle.




| Statements | Reasons |
|---|---|
| ▱GRAM; $\triangle GRM \cong \triangle RGA$ | Given |
| $\overline{RM} \cong \overline{GA}$ | CPCTC |
| ▱GRAM is a rectangle | If the diagonals of a parallelogram are congruent the parallelogram is a rectangle |

9. Summarize this part of the activity by making a list of the ways to prove that a quadrilateral (or parallelogram) is a rectangle.

Show that a quadrilateral has four right angles. Show that a parallelogram has one right angle. Show that a quadrilateral is equiangular. Show that the diagonals of a parallelogram are congruent.

## Check Your Understanding

10. Jamie says a quadrilateral with one right angle is a rectangle. Find a counterexample to show that Jamie is incorrect.

11. Do the diagonals of a rectangle bisect each other? Justify your answer.

## LESSON 16-2 PRACTICE

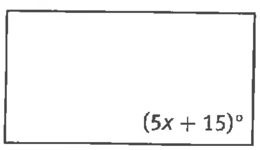
Three vertices of a rectangle are given. Find the coordinates of the fourth vertex.

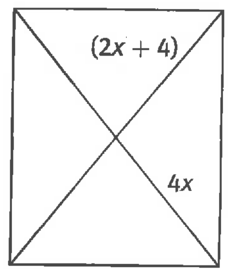
12. $(-3, 2), (-3, -1), (3, -1)$

13. $(-12, 2), (-6, -6), (4, 2)$

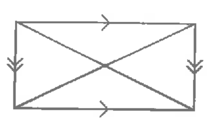
14. $(4, 5), (-3, -4), (6, -1)$

Find the value of $x$ that makes the parallelogram a rectangle.

15.



16.



17. **Model with mathematics.** Jill is building a new gate for her yard as shown. How can she use the diagonals of the gate to determine if the gate is a rectangle?





My Notes

10) Sample answer:

11) Yes. The diagonals are congruent and the diagonals bisect each other.

12) (3,2)

13) (-2,10)

14) (-5, 2)

15) $5x+15=90$
$5x=75$
$x=15$

16) $4x=2x+4$
$2x=4$
$x=2$

17) Jill can measure the lengths of the diagonals. If their measures are equal, then the gate is a rectangle.

My Notes

## Learning Targets:

- Develop criteria for showing that a quadrilateral is a rhombus.
- Prove that a quadrilateral is a rhombus.

**SUGGESTED LEARNING STRATEGIES:** Think-Pair-Share, Create Representations, Group Presentation, Discussion Groups

**1.** Complete the following definition.

A rhombus is a parallelogram with four congruent sides

**2. a.** Complete the theorem.

**Theorem** If a parallelogram has two consecutive congruent sides, then it has four congruent sides, and it is a rhombus.

**b.** Use one or more properties of a parallelogram and the definition of a rhombus to explain why the theorem in Item 2a is true.

Opposite sides of a parallelogram are congruent, so all sides of this para are congruent. By definition, this parallelogram is a rhombus.

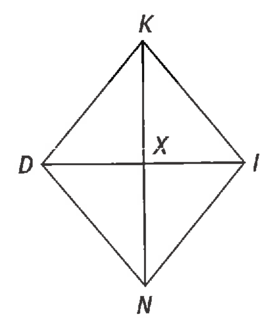
**3.** Complete the theorem.

**Theorem** If a quadrilateral is equilateral, then it is a rhombus

**4.** Write a paragraph proof to explain why the theorem in Item 3 is true.

If all sides of a quadrilateral are congruent, both pairs of opposite sides are congruent. Hence, the quad is a parallelogram. By definition, the parallelogram is a rhombus

**5.** Given $\square KIND$ with $\overline{KN} \perp \overline{ID}$.




**a.** List the three triangles that are congruent to $\triangle KXD$, and give the reason for the congruence.

$\triangle KXD \cong \triangle KXI \cong \triangle NXI \cong \triangle NXD$ by SAS

**b.** List all segments congruent to $\overline{KD}$ and explain why.

$\overline{KD} \cong \overline{KI} \cong \overline{NI} \cong \overline{ND}$ because of CPCTC

**c.** Complete the theorem.

**Theorem** If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus

**6.** Given $\square BIRD$ with coordinates $B(-2, -3)$, $I(1, 1)$, $R(6, 1)$, and $D(3, -3)$.

**a.** Show that $\square BIRD$ is a parallelogram.

Method 1: Both pairs of opp sides parallel
(Slope of $\overline{BI}$ and $\overline{RD} = \frac{4}{3}$; slope of $\overline{IR}$ and $\overline{BD} = 0$)

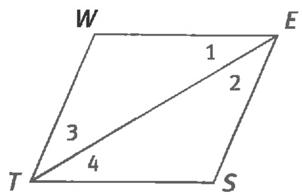
Method 2: Both pairs of opp sides congruent
$BI = RD = 5$; $IR = BD = 5$

Method 3: 1 pair of opp sides $\cong$ and $\parallel$
slope of $\overline{BI}$ = slope of $\overline{RD} = \frac{4}{3}$ and $BI = RD = 5$

**b.** Use the theorem in Item 5 to show $\square BIRD$ is a rhombus.

Diagonals are perpendicular: slope of $\overline{BR} = \frac{1}{2}$; slope of $\overline{ID} = -2$

**7.** Given $\square WEST$ with $\overline{TE}$ that bisects $\angle WES$ and $\angle WTS$.




**a.** List all angles congruent to $\angle 1$ and explain why.

$\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$; $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ (def. of $\angle$ bisector)
and $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ (alt. int $\angle$s and def. of parallelogram)

**b.** In $\triangle WET$, $\overline{WT} \cong$ <u>$\overline{WE}$</u>. In $\triangle SET$, $\overline{ST} \cong$ <u>$\overline{SE}$</u>. Explain.

If two angles of a triangle are congruent, the sides opposite those angles are congruent.

**c.** Complete the theorem.

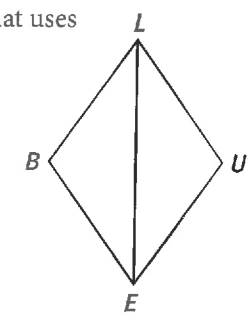
**Theorem** If a diagonal bisects <u>a pair of opposite angles</u> in a parallelogram, then the parallelogram is a <u>rhombus</u>.

**8. Construct viable arguments.** Write a proof that uses the theorem in Item 7 as the last reason.

**Given:** $\square BLUE$

$\triangle BLE \cong \triangle ULE$

**Prove:** $\square BLUE$ is a rhombus.




| Statements | Reasons |
|---|---|
| $\triangle BLE \cong \triangle ULE$ | Given |
| $\angle BLE \cong \angle ULE$; $\angle BEL \cong \angle UEL$ | CPCTC |
| $\overline{LE}$ bisects $\angle BLU$ and $\angle BEU$ | Def. of bisector |
| $\square BLUE$ | Given |
| $\square BLUE$ is a rhombus | If a diagonal bisects a pair of opp $\angle$s in a parallelogram, the parallelogram is a rhombus. |

My Notes

Method 4: Diagonals bisect each other
midpoint of $\overline{BR}$ = midpoint of $\overline{ID}$ = $(2, -1)$

My Notes

**9.** Summarize this part of the activity by making a list of the ways to prove that a quadrilateral is a rhombus.

Show that a quad has four sides; show that a para has two consecutive congruent sides; show that that the diagonals of a para are perpendicular; Show that a diagonal of a para bisects a pair of opposite angles.

**Check Your Understanding**

**10.** Can a rectangle ever be classified as a rhombus as well? Explain.

Yes. If a rectangle is a square, it is a rhombus.

## LESSON 16-3 PRACTICE

Three vertices of a rhombus are given. Find the coordinates of the fourth vertex.

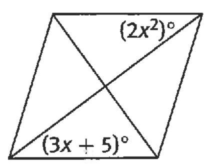
**11.** $(-2, -8)$, $(3, -3)$, $(-9, -7)$

**12.** $(-1, 2)$, $(-1, -1)$, $(2, 1)$

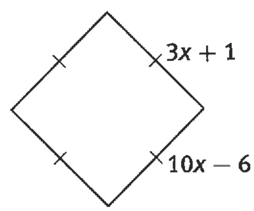
**13.** $(1, 1)$, $(-1, -2)$, $(1, -5)$

**14.** Find the value of $x$ that makes the parallelogram a rhombus.

a.




b.




**15.** **Reason quantitatively.** LaToya is using a coordinate plane to design a new pendant for a necklace. She wants the pendant to be a rhombus. Three of the vertices of the rhombus are $(3, 1)$, $(-1, -1)$, and $(1, -2)$. Assuming each unit of the coordinate plane represents one centimeter, what is the perimeter of the pendant? Round your answer to the nearest tenth.

11) $(-4, -2)$

12) $(2, -2)$

13) $(3, -2)$

14a) $2x^2 = 3x + 5$

$2x^2 - 3x - 5 = 0$

$x^2 - 3x - 10 = 0$

$(x - \frac{5}{2})(x + \frac{2}{2}) = 0$

$\boxed{x = \frac{5}{2}}$ $\quad x = -1$

14b) $10x - 6 = 3x + 1$

$7x = 7$

$x = 1$

15) 14.4cm

My Notes

## Learning Targets:

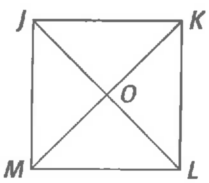
- Develop criteria for showing that a quadrilateral is a square.
- Prove that a quadrilateral is a square.

**SUGGESTED LEARNING STRATEGIES:** Think-Pair-Share, Create Representations, Group Presentation, Discussion Groups

**1.** Given $\square JKLM$.

**a.** What information is needed to prove that $\square JKLM$ is a square?

All four sides are congruent and four right angles or the diagonals bisect each other and the diagonals are congruent and perpendicular.




**b.** What additional information is needed to prove that $\square JKLM$ is a square? Explain.

Consecutive sides are congruent and one right angle or the diagonals are congruent and perpendicular.

**c.** What additional information is needed to prove that rectangle $JKLM$ is a square? Explain.

Consecutive sides are congruent or the diagonals are perpendicular.

**d.** What additional information is needed to prove that rhombus $JKLM$ is a square? Explain.

There is one right angle or the diagonals are congruent

**2.** Given $\square DAVE$ with coordinates $D(-1, 1)$, $A(0, 7)$, $V(6, 6)$, and $E(5, 0)$. Show that $\square DAVE$ is a square. One of the following.

| Method | Evidence |
|---|---|
| All sides are congruent and consecutive sides are perpendicular | $DA = AV = VE = DE = \sqrt{37}$<br>slope of $\overline{DA}$ = slope of $\overline{VE} = 6$<br>slope of $\overline{AV}$ = slope of $\overline{DE} = -\frac{1}{6}$ |
| Diagonals bisect each other and diagonals are perpendicular and congruent. | midpoint of $\overline{DV}$ = midpoint of $\overline{AE} = (2.5, 3.5)$<br>slope of $\overline{DV} = \frac{5}{7}$; slope of $\overline{AE} = -\frac{7}{5}$<br>$AE = DV = \sqrt{74}$ |

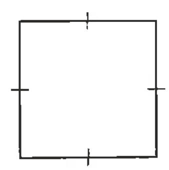
My Notes

5) No
6) Yes
7) No
8) Yes
9) $s\sqrt{2}$

3. **Critique the reasoning of others.** Several students in a class made the following statements. Decide whether you agree with each statement. If you disagree, change the statement to make it correct.
   a. A quadrilateral with congruent diagonals must be a rectangle.
   Disagree: A parallelogram with congruent diagonals must be a rectangle
   b. A parallelogram with two right angles must be a square.
   Disagree: A parallelogram with two right angles must be a parallelogram
   c. A quadrilateral with a pair of opposite parallel sides is always a parallelogram.
   Disagree: A quadrilateral with two pairs of opposite parallel sides is always a parallelogram
   d. A rhombus with four congruent angles is a square.
   agree

## Check Your Understanding

4. Elena has a garden with congruent sides, as shown below. Describe two different ways to show the garden is square.



→Show the diagonals are congruent
→show the figure has one right angle

## LESSON 16-4 PRACTICE

The coordinates of a parallelogram are given. Determine whether the figure is a square.

5. (−2, 3), (3, 3), (3, 0), (−2, 0)
6. (0, 1), (−1, 3), (1, 4), (2, 2)
7. (3, 6), (6, 2), (−2, 3), (−5, 7)
8. (3, 8), (−1, 6), (1, 2), (5, 4)
9. **Express regularity in repeated reasoning.** Find the length of the diagonal of a square with three of its vertices at (1, 0), (0, 0), and (0, 1). Then find the length of the diagonal of a square with three of its vertices at (2, 0), (0, 0), and (0, 2). Finally, find the length of the diagonal of a square with three of its vertices at (3, 0), (0, 0), and (0, 3). Use your findings to make a conjecture about the length of the diagonal of a square with three of its vertices at ($s$, 0), (0, 0), and (0, $s$).