Three-Dimensional Figures

ACTIVITY 33

What's Your View?

Lesson 33-1 Prisms and Pyramids

My Notes

Learning Targets:

- Describe properties and cross sections of prisms and pyramids.
- Describe the relationship among the faces, edges, and vertices of a polyhedron.

SUGGESTED LEARNING STRATEGIES: Create Representations, Use Manipulatives, Self Revision/Peer Revision, Interactive Word Wall, Visualization

Len Oiler has gone into business for himself. He designs and builds tents for all occasions. Len usually builds a model of each tent before he creates the actual tent. As he builds the models, he thinks about two things: the frame and the fabric that covers the frame. The patterns for Len's first two designs are shown by the *nets* below.

1. Build a frame for each model.

Figure 1

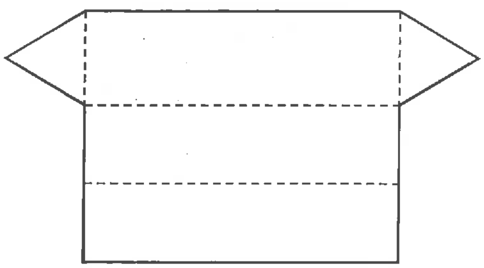
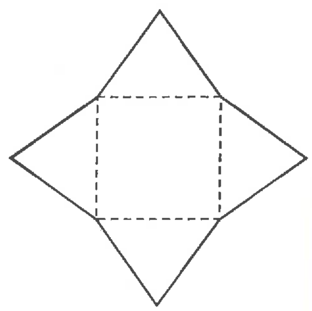
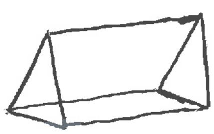
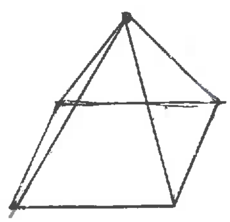


Figure 2







MATH TERMS

A **net** is a two-dimensional representation for a three-dimensional figure.

Faces are the polygons that make up a three-dimensional figure.

An **edge** is the intersection of any two faces.

A **vertex** is a point at the intersection of three or more faces.

In an **oblique prism**, the edges of the faces connecting the bases are not perpendicular to the bases. In a **right prism**, those edges are perpendicular to the bases.

2. A *prism* is a three-dimensional solid with two congruent, polygonal, and parallel bases, whose other *faces* are called lateral faces. Knowing that prisms are named according to their base shape and whether they are right or oblique, refer to the first tent frame you constructed based on Figure 1.

a. What is the shape of the two congruent and parallel bases?

equilateral triangles

b. What is the shape of the three lateral faces?

rectangles

c. What is the best name for the solid formed by the net in Figure 1?

right triangular prism

d. Use geometric terms to compare and contrast right prisms and oblique prisms.

DISCUSSION GROUP TIP

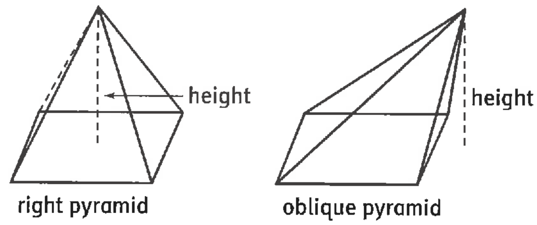
As you read and define new terms, discuss their meanings with other group members and make connections to prior learning.

My Notes

MATH TERMS

In an **oblique pyramid**, the intersection of the lateral faces (the top point of the pyramid) is not directly over the center of the base as it is in a right pyramid.

3. A ***pyramid*** is a three-dimensional solid with one polygonal base, whose lateral faces are triangles with a common vertex. Knowing that pyramids are named according to their base shape and whether they are right or oblique, refer to the second tent frame you constructed based on Figure 2 on the previous page.



a. What is the shape of the base?

square

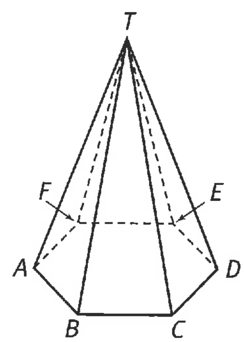
b. What is the best name for the solid formed by the net in Figure 2?

square right pyramid

c. Use geometric terms to compare and contrast right pyramids and oblique pyramids.

They both have triangular lateral faces. In a right pyramid, the height contains the center of the base.

4. The figure shown is a hexagonal pyramid. Locate the net for this solid on the worksheets your teacher has given you.



a. List the vertices of the pyramid.

A, B, C, D, E, F, T

b. List the base edges.

$\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{AF}$

c. List the lateral edges.

$\overline{AT}$, $\overline{BT}$, $\overline{CT}$, $\overline{DT}$, $\overline{ET}$, $\overline{FT}$

d. List the lateral faces.

$\triangle ABT$, $\triangle BCT$, $\triangle CDT$, $\triangle DET$, $\triangle EFT$, $\triangle AFT$

My Notes

5. Complete the table below. Nets for the pentagonal prism and pyramid can be found on the worksheets your teacher has given you. All listed figures are examples of polyhedra (singular: ***polyhedron***).

Figure Name	Number of Faces	Number of Vertices	Total Number of Edges
Triangular prism	5	6	9
Rectangular prism	6	8	12
Pentagonal prism	7	10	15
Square pyramid	5	5	8
Pentagonal pyramid	6	6	10
Hexagonal pyramid	7	7	12
Hexagonal prism	8	12	18
Triangular pyramid	4	4	6

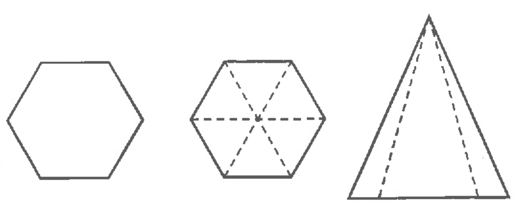
6. List any patterns that you observe in the table above.

7. **Make use of structure.** Write a rule that expresses the relationship among F, V, and E.

$F+V=E+2$

8. Three views of the hexagonal pyramid are shown below.
   a. Label each view as *side*, *top*, or *bottom*.

from left to right: bottom, top, side



   b. **Attend to precision.** Explain the significance of the dashed segments in the views.

The dashed segments represent edges visible to the viewer.

MATH TERMS

A **polyhedron** is a three-dimensional figure consisting of polygons that are joined along their edges.

In any polyhedron, the intersection of any two faces is called an **edge**, and the vertices of the faces are the **vertices of the polyhedron**.

DISCUSSION GROUP TIPS

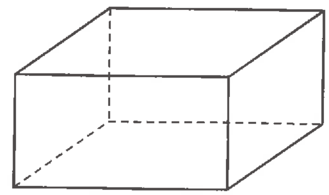
In your discussion groups, read the text carefully to clarify meaning. Reread definitions of terms as needed to help you comprehend the meanings of words, or ask your teacher to clarify vocabulary words.

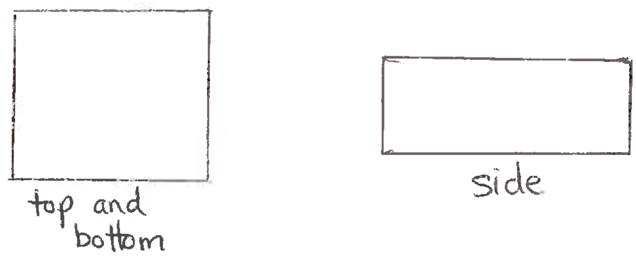
CONNECT TO HISTORY

Leonard Euler (1707–1783) was one of the first mathematicians to write about the relationship between the number of faces, vertices, and edges in polyhedrons. Leonard Euler was not just a "one-dimensional" man; he made contributions to the fields of physics and shipbuilding, too.

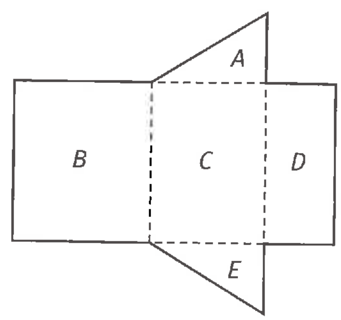
My Notes

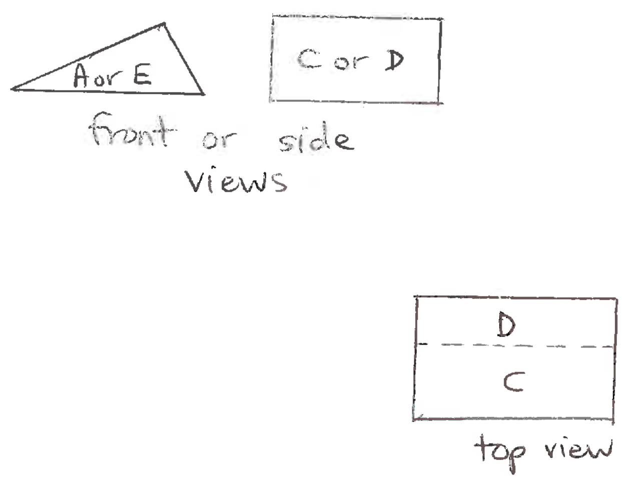
9. Sketch and label the bottom, top, and side views of this rectangular prism.





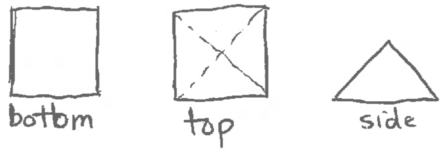
10. If the bottom of the solid formed by the net shown is face B, sketch the top, front, and side views.



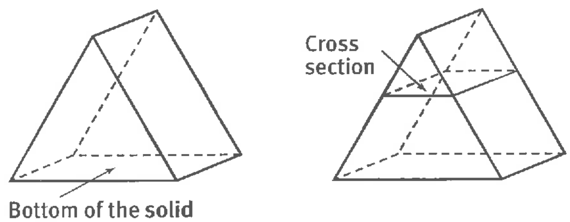


My Notes

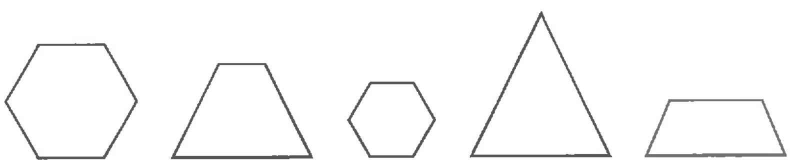
11. Sketch the bottom, top, and side views of a square pyramid.



A ***cross section*** of a solid figure is the intersection of that figure and a plane.



12. Make sense of problems. Below are several cross sections of a hexagonal right pyramid. Label each cross section as being *parallel* or *perpendicular* to the base of the pyramid.



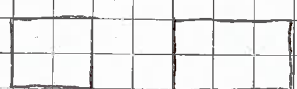
in order from left to right: parallel, perpendicular, parallel, perpendicular, perpendicular

My Notes

14) Both have polygonal bases and can be used in Euler's Formula for the number of faces, edges, and vertices. Prisms have two bases and quadrilateral faces. Pyramids have one base and triangular lateral faces

15a) parallel to base

15b) perpendicular to base



15c) The cross sections parallel to the base are all congruent triangles. The cross sections perpendicular to the base are all rectangles with the same height as the prism and varying widths.

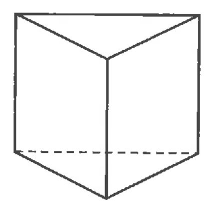
13. Sketch several cross sections that are parallel to the base of a square pyramid. Sketch several cross sections that are perpendicular to the base of a square pyramid. What patterns do you observe?

Cross sections parallel to the base are always squares. Cross sections perpendicular to the base are triangles and trapezoids.

Check Your Understanding

14. Use geometric terms to compare and contrast prisms and pyramids.

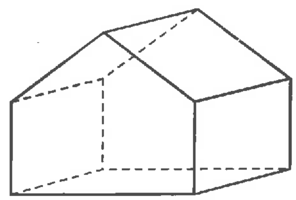
15. **Model with mathematics.** Consider the triangular prism below.



a. Sketch several cross sections that are parallel to the bases of the triangular prism.
b. Sketch several cross sections that are perpendicular to the bases of the prism.
c. Describe the patterns you notice.

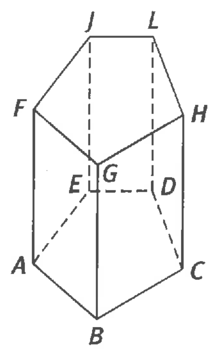
LESSON 33-1 PRACTICE

16. Consider the prism below.



a. Create a net for the figure.
b. Sketch and label the top, bottom, and side views of the figure.

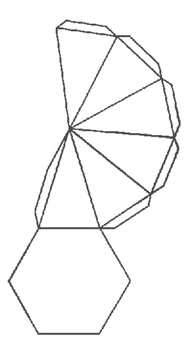
17. Consider the polyhedron below.



a. Identify the vertices of the figure.
b. Identify the shape of the two congruent and parallel bases.
c. Identify the shape of the lateral faces.
d. How many lateral edges compose the figure?
e. Name each lateral face.
f. List the base edges.
g. What is the best name for the polyhedron?

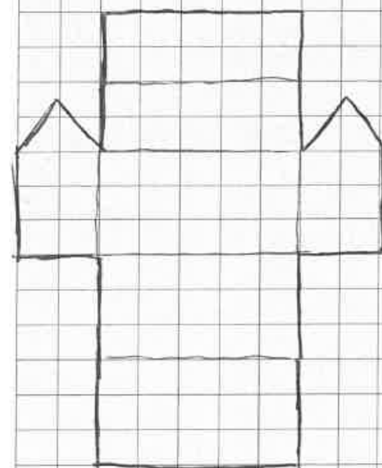
18. A polyhedron has 10 edges. Explain how to determine the sum of the number of faces and vertices that compose the figure. Then name the polyhedron.

19. **Critique the reasoning of others.** The design for a new toy is shown in the net below. Jamal classifies the toy as a hexagonal prism since there are six lateral faces that meet at one point. Do you agree with Jamal? Justify your response.



My Notes

16a) Sample sketch



b) top bottom side

side

17a) A, B, C, D, E, F, G, H, I, J
17b) pentagon
17c) rectangles
17d) 5
17e) rectangles ABGF, BCHG, CDIH, DGJI, EAFJ
17f) $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{FG}$, $\overline{GH}$, $\overline{HI}$, $\overline{IJ}$, $\overline{JF}$
17g) pentagonal prism
18) It is a pentagonal pyramid; use Euler's Formula; $F+V=E+2$
$F+V=12$
19) No, the toy is a hexagonal pyramid. The figure is a pyramid, not a prism; there is only one base.

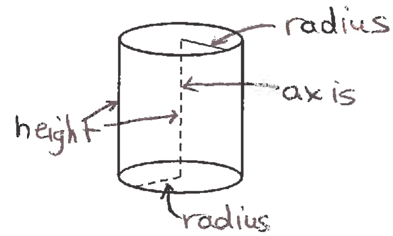
My Notes

Learning Targets:

- Describe properties and cross sections of a cylinder.
- Describe properties and cross sections of a cone.

SUGGESTED LEARNING STRATEGIES: Group Presentation, Self Revision/Peer Revision, Visualization, Think-Pair-Share, Interactive Word Wall

A ***cylinder*** is the set of all points in space that are a given distance (radius) from a line known as the axis. Bases are formed by the intersection of two parallel planes with the cylinder. If the axis is perpendicular to each of the bases, the cylinder is called a right cylinder.

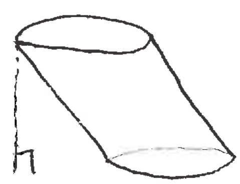


1. Label the axis, radius, and height in the diagram of the right cylinder.

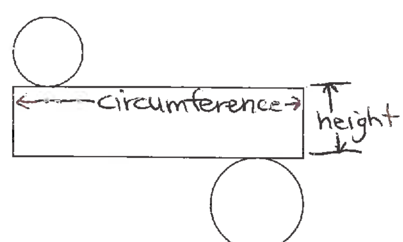
2. Describe the shape and size of the cross sections that are parallel to the bases.

   The cross sections are congruent circles

3. **Attend to precision.** Sketch an oblique cylinder.



4. Len is working on a pattern for a right cylinder. Which base is the correct size for this cylinder? Explain your answer.



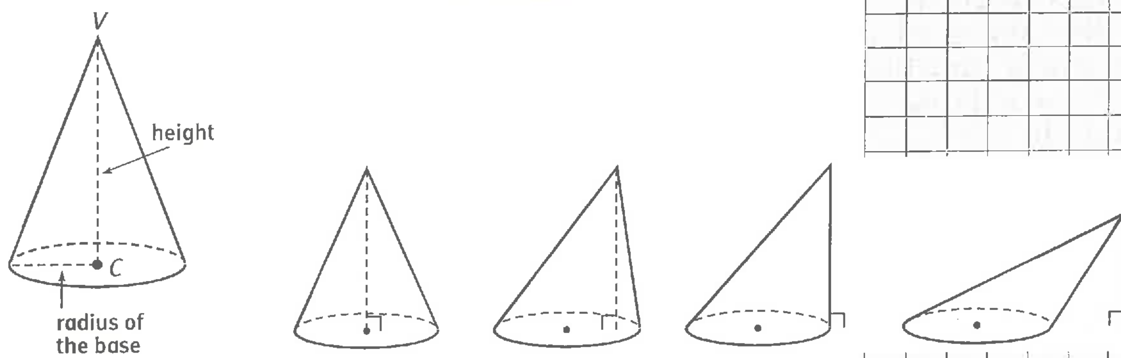
   The larger circle is the correct size, since the longest edge (length) of the lateral face is a little more than three times the diameter of the circle.

5. Explain how the dimensions of the rectangle in the net in Item 4 relate to the cylinder once it is constructed.

   The length of the rectangle equals the circumference of the base and the height of the rectangle equals the distance between the bases

My Notes

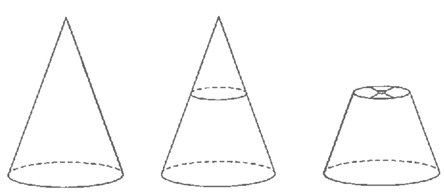
A **cone** is the union of all segments in space that join points in a circle to a point, called the **vertex** of the cone, that is not coplanar with the circle. The base of a cone is the intersection of a cone and a plane. If the segment that joins the center of the circle to the vertex is perpendicular to the plane that contains the center, the cone is a **right cone** and this segment is called the ***height of the cone***. Otherwise, the cone is **oblique**.

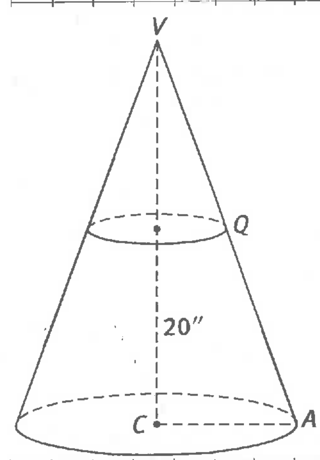


6. Compare and contrast the shape and size of the cross sections of a right cone that is parallel to the base with the cross sections of a cylinder that is parallel to its base.
The cross sections of a cone are circles (just as in the case of a cylinder), but they are not congruent.

7. **Reason abstractly.** Describe the cross section of a right cone that is perpendicular to the base and contains the vertex of the cone.
Sample answer: isosceles triangle. The height and radius form a right triangle with the lateral surface. Two coplanar right triangles are congruent by SAS, so the hypotenuses are congruent.

When the tent business gets slow, Len makes lampshades, which are formed when a plane parallel to the base intersects the cone.





8. Len knows the height of the cone is 30 inches and the radius is 9 inches. He needs to know the area and circumference of the smaller circle, which is the top of the shade. (Assume the cross sections are 20 inches apart and parallel.)
   a. Label the center of the small circle P, and draw in $\overline{PQ}$. Identify two similar triangles in the figure. Explain why they are similar.
   $\triangle VPQ$ and $\triangle VCA$
   b. Let r represent the radius of the small circle. Using corresponding sides of the similar triangles, write a proportion and solve for r.
   $\frac{r}{9} = \frac{(30-20)}{30}$; $r = 3$
   c. Find the circumference and area of the smaller cross section.
   $C = 2\pi(3) = 6\pi$ inches
   $A = \pi(3)^2 = 9\pi$ inches^2

My Notes

9) The cross sections are rectangles.

10) oblique cone

11) If the cross section intersects the axis at an angle, the cross section formed is an ellipse, not a circle.

12) Since a pipe is a cylinder, the cross-sectional area of a pipe is a circle. Use the formula $A = \pi r^2$, where $r = 2$ inches.

13a) rectangle

13b) 75.4 cm length by 15 cm height. Since the length of the rectangle equals the circumference of the base, use the radius of the can to find the circumference. The height of the rectangular label is the height of the can

13c) $(75.4)(15) = 1131 \text{ cm}^2$

14a) one axis of symmetry, infinite planes

14b) one axis of symmetry, infinite planes

14c) no axis of symmetry, one plane

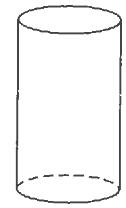
14d) no axis of symmetry, one plane

Check Your Understanding

9. Describe the shape of the cross sections that are perpendicular to the bases of a right cylinder.

10. A figure has one base. The distance from the center of the base to the vertex is greater than the height. What is the best name for the figure?

11. Emma and Malia are both studying cross sections of the right cylinder shown. Emma states that all planes that pass through the axis of the cylinder form cross sections in the shape of a circle. Malia says this is not always true. Provide a counterexample of Emma's statement to prove that Malia is correct.



LESSON 33-2 PRACTICE

12. Make sense of problems. PVC is a type of plastic that is often used in construction. A PVC pipe with a 4 in. diameter is used to transfer cleaning solution into a process vessel. Explain how to compute the cross-sectional area of the pipe.

13. A graphic designer is working on a new label for a product that is packaged in a can. The radius of the cylinder is 12 cm. The distance between the top and bottom of the can is 15 cm.

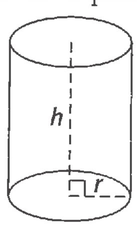
a. Describe the shape of the label prior to it being attached to the can.

b. What are the dimensions of the label so that it completely covers the curved area of the can? Explain how you determined your answer.

c. What is the area of the label, to the nearest square centimeter?

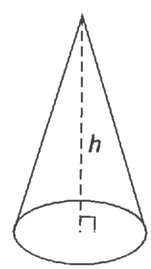
14. For each figure, determine the number of axes of symmetry and the number of planes of symmetry.

a.



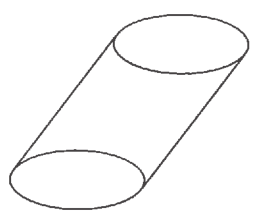
right cylinder

b.



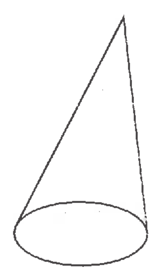
right cone

c.



oblique cylinder

d.



oblique cone

My Notes

Learning Targets:

- Describe properties and cross sections of a sphere.
- Identify three-dimensional objects generated by rotations of two-dimensional objects.

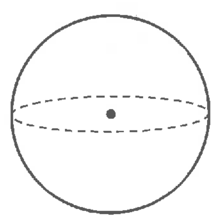
SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Vocabulary Organizer, Create Representations, Visualization, Think-Pair-Share

Len provides spherical-shaped paper lanterns for his customers to use on camping trips. A sphere is created when a circle is rotated in space about one of its diameters. A ***sphere*** is the set of all points in space that are a given distance (radius) from a given point (center).

MATH TERMS

The intersection of a plane through the center of a sphere is known as a **great circle**.

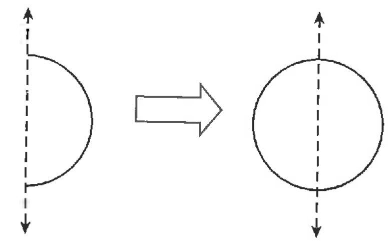
1. Consider the cross sections of a sphere.
   a. Describe the shape and size of the cross sections.
   The cross sections are circles of varying sizes.
   b. Where will the largest of these cross sections be located?
   The largest will occur when the plane that intersects the circle to form the cross section also contains the center of the sphere.
   c. Where is the center of the largest possible cross section of a sphere located in relation to the center of the sphere?
   The center of the great circle coincides with the center of the sphere.
2. The figure shown is a sphere.



   a. Write a definition for a *chord of a sphere*. Draw and label a chord in the diagram.
   A chord is a segment whose endpoints lie on the sphere.
   b. Write a definition for the *diameter of a sphere*. Draw and label a diameter in the diagram.
   A diameter is a chord that contains the center.
   c. Write a definition for a *tangent (line or plane) to a sphere*. Draw and label a tangent line and a tangent plane in the diagram.
   A tangent line (or plane) intersects the sphere at exactly one point.
   d. What must be true about a plane that is tangent to a sphere and a radius of the sphere drawn to the point of tangency? Explain your answer.
   The radius is perpendicular to the tangent plane at the point of tangency.

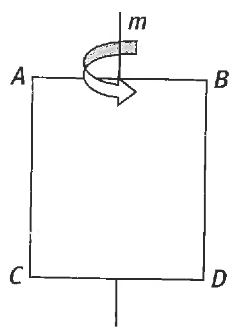
My Notes

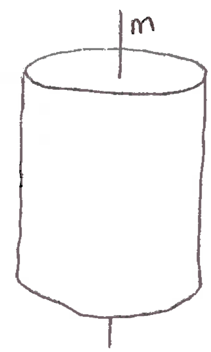
A sphere is created by rotating a semicircle about a line, as shown in the figure below.



Rotating a plane about a given axis such that it forms a three-dimensional figure is known as a ***solid of rotation***.

3. Model with mathematics. Sketch and name the solid generated by rotating rectangle $ABCD$ about line m.





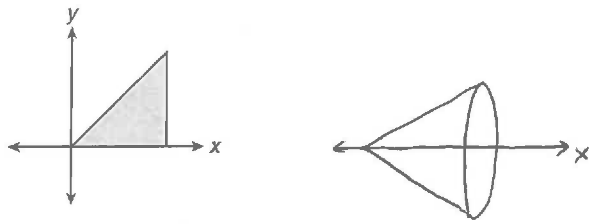
The rotation generated a cylinder.

CONNECT TO AP

Right cylinders, cones, and spheres are often called *solids of rotation* because they can be formed by rotating a rectangle, triangle, or semicircle around an axis. In calculus, you will learn how to compute the volume of solids of rotation using a definite integral.

My Notes

4. Sketch and name the solid generated by rotating the triangle about the x-axis.



The rotation generated a cone.

5. Describe how Figure B was created from Figure A. Then name the various three-dimensional figures that make up Figure B.

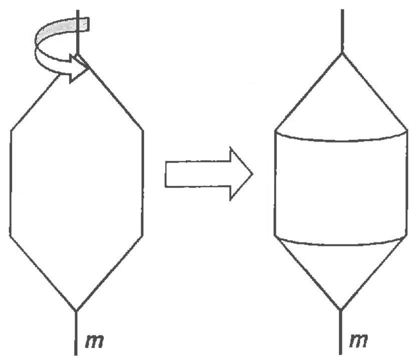


Figure A Figure B

Figure A was rotated about line m to generate Figure B. Figure B is composed of two cones and two cylinders.

6. **Use appropriate tools strategically.** Cut out any two-dimensional shape, tape it to a string or wooden stick, then hold the string/stick vertically and twirl it between your fingers to see the three-dimensional solid it forms.
   a. Sketch the three-dimensional figure formed by your model.
   b. Compare your model with a partner's. Describe the differences in the figures formed. What accounts for these differences?
   c. Describe other tools you can use to create a solid of rotation model. Use your model to generate at least two more solid figures.

My Notes

7) 1:1; the radius of the great circle is the same measure as the radius of the Sphere.

8) The intersection of the two spheres is a circle whose plane is perpendicular to the line joining the centers of the surfaces and whose center is on that line.

9) No; the figure formed would have to be symmetrical about the x-axis. So, it would not form an oblique cone. It would form a right cone. The axis of symmetry is the x-axis.

10) The areas of the circles get smaller.

11) The diameter of a sphere is the greatest chord of the sphere.

12) cylinder

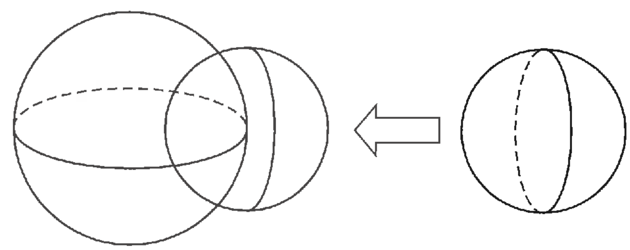
13) B

14) $2\sqrt{2} \approx 2.8$ cm

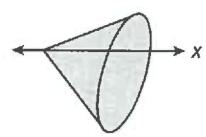
Check Your Understanding

7. What is the ratio of the measure of the radius of a sphere to the measure of the radius of its great circle? Explain.

8. Two spheres intersect, as shown in the diagram below. Describe the intersection of the two spheres.



9. Can a solid of rotation about the x-axis generate the three-dimensional figure as shown? Explain.

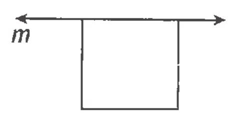


LESSON 33-3 PRACTICE

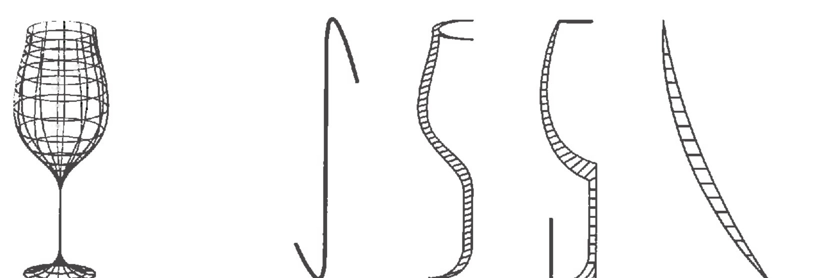
10. How do the areas of the cross sections of a sphere compare to each other as you move farther away from the center of the sphere?

11. Identify by name the greatest chord in a sphere.

12. What figure is formed by rotating a square about line m?



13. Which of the following two-dimensional figures was rotated about the y-axis to form the three-dimensional figure shown?



A. B. C. D.

14. **Model with mathematics.** Keiko designed a pendant for a necklace by rotating a square about a diagonal, as shown. What is the width of the pendant at its widest point?