

Activity 34

KEY

Volume of Prisms and Cylinders

The volume of a solid is the number of cubic units needed to fill up its interior completely. A cubic unit is a cube whose sides measure one unit.

Volume of Prisms

$$\text{Volume} = \frac{Bh}{1}$$

↑ "B" is replaced by an area formula

$$B = \frac{\text{area of the base}}{1}$$

$$h = \frac{\text{height of the prism}}{1}$$

The "B" is replaced by the appropriate area formula that corresponds to the shape of the base. The height of the prism is the perpendicular distance between the bases.

These formulas apply to all prisms, such as triangular prisms, rectangular prisms, rhombic prisms, pentagonal prisms, hexagonal prisms, trapezoidal prisms, etc.

Postulates

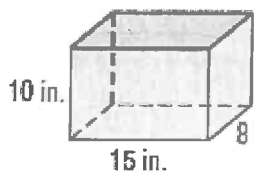
Volume Postulate: Every solid has a volume given by a positive real number.

Congruent Solids Postulate: Congruent solids have the same volume.

Volume Addition Postulate: If the interiors of two solids do not intersect, then the volume of their union is the sum of their volumes.

Examples: Find the volume of each prism.

1)



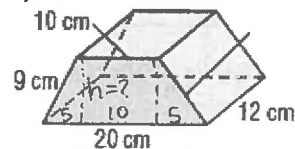
$$V = Bh$$

$$V = lwh$$

$$V = (15)(8)(10)$$

$$V = 1200 \text{ in}^3$$

2)



$$h^2 = 9^2 - 5^2$$

$$h^2 = 81 - 25$$

$$h = \sqrt{56}$$

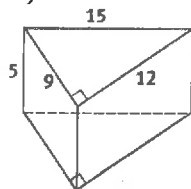
$$V = Bh$$

$$V = \left(\frac{1}{2}(b_1 + b_2)(h)\right)(l)$$

$$V = \frac{1}{2}(10 + 20)(\sqrt{56})(12)$$

$$V = 1347 \text{ cm}^3$$

3)



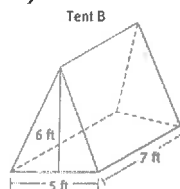
$$V = Bh$$

$$V = \left(\frac{1}{2}bh\right)(h)$$

$$V = \left(\frac{1}{2}\right)(12)(9)(5)$$

$$V = 270 \text{ in}^3$$

4)



$$V = Bh$$

$$V = \left(\frac{1}{2}bh\right)(h)$$

$$V = \left(\frac{1}{2}\right)(5)(6)(7)$$

$$V = 105 \text{ ft}^3$$

5) The length of a rectangular solid is twice the sum of the width and height of the rectangular solid. If the width is equal to the height and the volume of the solid is 108 in^3 , what is the length of the solid?

$$l = 2(w+h)$$

$$w = h$$

$$V = lwh$$

$$V = 2(w+h)(w)(h)$$

$$V = 2(2w)(w)(w)$$

$$108 = 4w^3$$

$$27 = w^3$$

$$w = 3 \text{ inches}$$

$$l = 2(3+3)$$

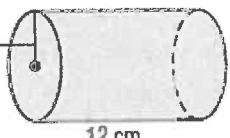
$$l = 12 \text{ inches}$$

Volume of Cylinders

$$\text{Volume} = \underline{Bh \text{ or } \pi r^2 h}$$

Examples: Find the volume of each cylinder.

1)



$r = 4 \text{ cm}$
 $h = 12 \text{ cm}$

$$V = \pi r^2 h$$

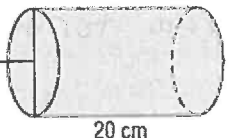
$$V = \pi (4)^2 (12)$$

$$V = 192\pi \text{ cm}^3$$

OR

$$603.186 \text{ cm}^3$$

2)



$r = 4 \text{ cm}$
 $h = 20 \text{ cm}$

$$V = \pi r^2 h$$

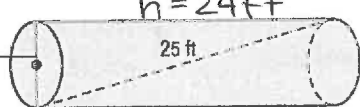
$$V = \pi (4)^2 (20)$$

$$V = 320\pi \text{ cm}^3$$

OR

$$1005.31 \text{ cm}^3$$

3)



$h = 24 \text{ ft}$
 25 ft

$$h^2 = 25^2 - 7^2$$

$$h^2 = 625 - 49$$

$$h^2 = 576$$

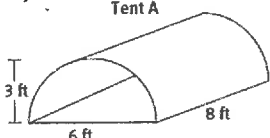
$$h = 24 \text{ ft}$$

$$V = \pi r^2 h$$

$$V = \pi (3.5)^2 (24)$$

$$V = 294\pi \text{ or } 923.628 \text{ ft}^3$$

4)



Tent A
 3 ft
 6 ft
 8 ft
 $r = 3 \text{ ft}$
 $h = 8 \text{ ft}$

$$V = \frac{\pi r^2 h}{2}$$

$$V = \frac{\pi (3)^2 (8)}{2} = \frac{72\pi}{2}$$

$$V = 36\pi \text{ or } 113.1 \text{ ft}^3$$

5) The volume of a cylinder is 220 cm^3 , and the radius is 5 cm . What is the height of the cylinder, to the nearest tenth?

$$V = \pi r^2 h$$

$$220 = \pi (5)^2 (h)$$

$$220 = (25\pi)(h)$$

$$h = \frac{220}{25\pi}$$

$$h = 2.8 \text{ cm}$$

6) The radius of a cylinder is $2x$ and the height of the cylinder is $8x + 2$. What is the volume of the cylinder in terms of x ?

$$V = \pi r^2 h$$

$$V = \pi (2x)^2 (8x + 2)$$

$$V = \pi (4x^2) (8x + 2)$$

$$V = (32x^3 + 8x^2)\pi \text{ in}^3$$