

# Proofs about Line Segments and Angles

## Now I'm Convinced

### Lesson 6-1 Justifying Statements

#### Learning Objectives

- Use definitions, properties, and theorems to justify a statement.
- Write two-column proofs to prove theorems about lines and angles.

#### Learning Target:

- Use properties, postulates, and definitions to justify statements.

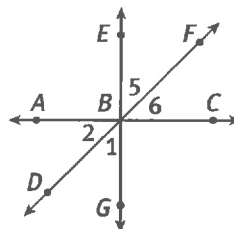
**SUGGESTED LEARNING STRATEGIES:** Close Reading, Activating Prior Knowledge, Think-Pair-Share, Discussion Groups

A proof is an argument, a justification, or a reason that something is true. A proof is an answer to the question “why?” when the person asking wants an argument that is indisputable.

There are three basic requirements for constructing a good proof.

- Awareness and knowledge of the definitions of the terms related to what you are trying to prove.
- Knowledge and understanding of postulates and previous proven theorems related to what you are trying to prove.
- Knowledge of the basic rules of logic.

To write a proof, you must be able to justify statements. The statements in Example A are based on the diagram to the right in which lines  $AC$ ,  $EG$ , and  $DF$  all intersect at point  $B$ . Each of the statements is justified using a property, postulate, or definition.



#### Example A

Name the property, postulate, or definition that justifies each statement.

Statement	Justification
a. If $\angle ABE$ is a right angle, then $m\angle ABE = 90^\circ$ .	Definition of right angle
b. If $\angle 2 \cong \angle 1$ and $\angle 1 \cong \angle 5$ , then $\angle 2 \cong \angle 5$ .	Transitive Property
c. Given: $B$ is the midpoint of $\overline{AC}$ . Prove: $\overline{AB} \cong \overline{BC}$	Definition of midpoint
d. $m\angle 2 + m\angle ABE = m\angle DBE$	Angle Addition Postulate
e. If $\angle 1$ is supplementary to $\angle FBG$ , then $m\angle 1 + m\angle FBG = 180^\circ$ .	Definition of supplementary angles

#### MATH TIP

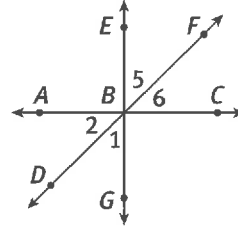
The Reflexive, Symmetric, and Transitive Properties apply to congruence as well as to equality.

My Notes

My Notes

**Try These A**

Using the diagram from the previous page, reproduced here, name the property, postulate, or definition that justifies each statement.



Statement	Justification
a. $EB + BG = EG$	Segment Addition Postulate
b. If $\angle 5 \cong \angle 6$ , then $\overline{BF}$ bisects $\angle EBC$ .	Definition of angle bisector
c. If $m\angle 1 + m\angle 6 = 90^\circ$ , then $\angle 1$ is complementary to $\angle 6$ .	Definition of complementary angles
d. If $m\angle 1 + m\angle 5 = m\angle 6 + m\angle 5$ , then $m\angle 1 = m\angle 6$ .	Subtraction Property of Equality
e. Given: $\overline{AC} \perp \overline{EG}$ Prove: $\angle ABG$ is a right angle.	Definition of perpendicular lines

**Lesson 6-1**  
**Justifying Statements**

**ACTIVITY 6**

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**Check Your Understanding**

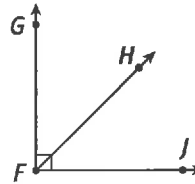
1. Explain why the following statement does not need to be justified.

*The midpoint of a segment is a point on the segment that divides it into two congruent segments.*

2. Given:  $\overline{RS}$  and  $\overline{ST}$  share endpoint  $S$ .

**Critique the reasoning of others.** Based on this information, Michaela says that the Segment Addition Postulate justifies the statement that  $RS + ST = RT$ . Is there a flaw in Michaela's reasoning, or is she correct? Explain.

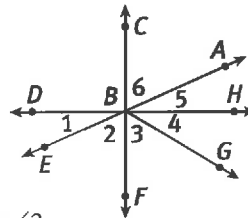
3. **Construct viable arguments.** Write and justify two statements based on the information in the figure.



**LESSON 6-1 PRACTICE**

Lines  $CF$ ,  $DH$ , and  $EA$  intersect at point  $B$ . Use this figure for Items 4–8. Write the definition, postulate, or property that justifies each statement.

4. If  $\angle 2$  is supplementary to  $\angle CBE$ , then  $m\angle 2 + m\angle CBE = 180^\circ$ .
5. If  $\angle 2 \cong \angle 3$ , then  $\overline{BF}$  bisects  $\angle GBE$ .
6.  $CB + BF = CF$
7. If  $\angle DBF$  is a right angle, then  $\overline{HD} \perp \overline{CF}$ .
8. If  $m\angle 3 = m\angle 6$ , then  $m\angle 3 + m\angle 2 = m\angle 6 + m\angle 2$ .
9. **Reason abstractly.** Write a statement related to the figure above that can be justified by the Angle Addition Postulate.



- 4) Definition of supplementary angles
- 5) Definition of angle bisector
- 6) Segment Addition Postulate
- 7) Definition of perpendicular lines
- 8) Addition Property of Equality
- 9) Sample answer:  $m\angle EBC = m\angle EBD + m\angle DBC$

**My Notes**

1) The statement is the definition, and a definition does not need to be proven or justified.

**MATH TIP**

Do not assume that an angle is a right angle just because it appears to measure  $90^\circ$ . You can only conclude that an angle is a right angle if (1) you are given this information, (2) a diagram of the angle includes a right angle symbol, or (3) you prove that the angle is a right angle.

2) It is not known if point S is between points R and T.



3)  $\angle GFJ$  measures  $90^\circ$   
Definition of a right angle

$m\angle GFJ = m\angle GFH + m\angle HFT$   
Angle Addition Postulate

My Notes

**MATH TERMS**

The **Vertical Angles Theorem** states that vertical angles are congruent.

**DISCUSSION GROUP TIPS**

As you listen to the group discussion, take notes to aid comprehension and to help you describe your own ideas to others in your group. Ask questions to clarify ideas and to gain further understanding of key concepts.

**Learning Targets:**

- Complete two-column proofs to prove theorems about segments.
- Complete two-column proofs to prove theorems about angles.

**SUGGESTED LEARNING STRATEGIES:** Vocabulary Organizer, Think-Pair-Share, Close Reading, Discussion Groups, Self Revision/Peer Revision, Group Presentation

Earlier, you wrote two-column proofs to solve algebraic equations. You justified each statement in these proofs by using an algebraic property. Now you will use two-column proofs to prove geometric theorems. You must justify each statement by using a definition, a postulate, a property, or a previously proven theorem.

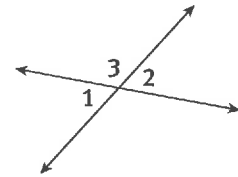
Recall that *vertical angles* are opposite angles formed by a pair of intersecting lines. In the figure below,  $\angle 1$  and  $\angle 2$  are vertical angles. The following example illustrates how to prove that vertical angles are congruent.

**Example A**

*Theorem: Vertical angles are congruent.*

**Given:**  $\angle 1$  and  $\angle 2$  are vertical angles.

**Prove:**  $\angle 1 \cong \angle 2$



Statements	Reasons
1. $m\angle 1 + m\angle 3 = 180^\circ$	1. Definition of supplementary angles
2. $m\angle 2 + m\angle 3 = 180^\circ$	2. Definition of supplementary angles
3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	3. Substitution Property
4. $m\angle 1 = m\angle 2$	4. Subtraction Property of Equality
5. $\angle 1 \cong \angle 2$	5. Definition of congruent angles

**Guided Example B**

Supply the missing statements and reasons.

*Theorem: All right angles are congruent.*

**Given:**  $\angle A$  and  $\angle B$  are right angles.

**Prove:**  $\angle A \cong \angle B$

Statements	Reasons
1. $\angle A$ and $\angle B$ are right angles	1. Given
2. $m\angle A = 90^\circ$ ; $m\angle B = 90^\circ$	2. Definition of right angles
3. $m\angle A = m\angle B$	3. Transitive Property
4. $\angle A \cong \angle B$	4. Definition of congruent angles

**Lesson 6-2**  
**Two-Column Geometric Proofs**

**ACTIVITY 6**

*continued*

My Notes

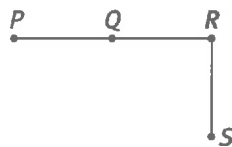
**Try These A-B**

a. Complete the proof.

Given:  $Q$  is the midpoint of  $\overline{PR}$ .

$\overline{QR} \cong \overline{RS}$

Prove:  $\overline{PQ} \cong \overline{RS}$

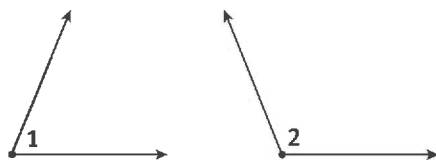


Statements	Reasons
1. $Q$ is the midpoint of $\overline{PR}$ .	1. Given
2. $\overline{PQ} \cong \overline{QR}$	2. Definition of midpoint
3. $\overline{QR} \cong \overline{RS}$	3. Given
4. $\overline{PQ} \cong \overline{RS}$	4. Transitive Property of Congruence

b. Complete the proof.

Given:  $\angle 1$  and  $\angle 2$  are supplementary.  
 $m\angle 1 = 68^\circ$

Prove:  $m\angle 2 = 112^\circ$



Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary.	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$	2. Definition of supplementary angles
3. $m\angle 1 = 68^\circ$	3. Given
4. $68^\circ + m\angle 2 = 180^\circ$	4. Substitution Property
5. $m\angle 2 = 112^\circ$	5. Subtraction Property of Equality

**MATH TIP**

More than one statement in a two-column proof can be given information.

Notice that "Given" doesn't only appear as the first Reason. It can be any reason except the last.

There may be more than one given statement.

My Notes

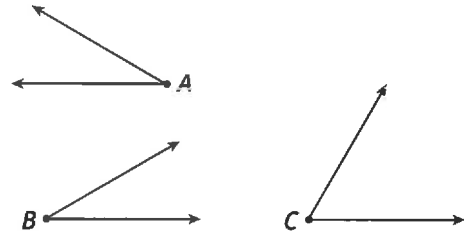
**MATH TIP**

The theorem stated in Example C is called the Congruent Complements Theorem.

**Example C**

Arrange the statements and reasons below in a logical order to complete the proof.

*Theorem: If two angles are complementary to the same angle, then the two angles are congruent.*



**Given:**  $\angle A$  and  $\angle B$  are each complementary to  $\angle C$ .

**Prove:**  $\angle A \cong \angle B$

$m\angle A + m\angle C = m\angle B + m\angle C$	Transitive Property
$\angle A$ and $\angle B$ are each complementary to $\angle C$ .	Given
$\angle A \cong \angle B$	Definition of congruent segments
$m\angle A = m\angle B$	Subtraction Property of Equality
$m\angle A + m\angle C = 90^\circ$ ; $m\angle B + m\angle C = 90^\circ$	Definition of complementary angles

Start the proof with the given information. Then decide which statement and reason follow logically from the first statement. Continue until you have proved that  $\angle A \cong \angle B$ .

Statements	Reasons
1. $\angle A$ and $\angle B$ are each complementary to $\angle C$ .	1. Given
2. $m\angle A + m\angle C = 90^\circ$ $m\angle B + m\angle C = 90^\circ$	2. Definition of complementary angles
3. $m\angle A + m\angle C = m\angle B + m\angle C$	3. Transitive Property
4. $m\angle A = m\angle B$	4. Subtraction Property of Equality
5. $\angle A \cong \angle B$	5. Definition of congruent angles



**Lesson 6-2**  
**Two-Column Geometric Proofs**

**ACTIVITY 6**  
*continued*

**Try These C**

a. **Attend to precision.** Arrange the statements and reasons below in a logical order to complete the proof.

**Given:**  $\angle 1$  and  $\angle 2$  are vertical angles;  $\angle 1 \cong \angle 3$ .

**Prove:**  $\angle 2 \cong \angle 3$

$\angle 1 \cong \angle 2$	Vertical angles are congruent.
$\angle 2 \cong \angle 3$	Transitive Property
$\angle 1 \cong \angle 3$	Given
$\angle 1$ and $\angle 2$ are vertical angles.	Given

b. Write a two-column proof of the following theorem.

**Theorem:** If two angles are supplementary to the same angle, then the two angles are congruent.

**Given:**  $\angle R$  and  $\angle S$  are each supplementary to  $\angle T$ .

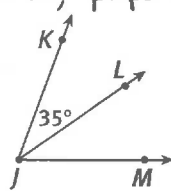
**Prove:**  $\angle R \cong \angle S$

**Check Your Understanding**

- If you know that  $\angle D$  and  $\angle F$  are both complementary to  $\angle J$ , what statement could you prove using the Congruent Complements Theorem?
- What types of information can you list as reasons in a two-column geometric proof? *Given information, postulates, properties, definitions, theorems*
- Kenneth completed this two-column proof. What mistake did he make? How could you correct the mistake?

**Given:**  $\overline{JL}$  bisects  $\angle KJM$ ;  $m\angle KJL = 35^\circ$

**Prove:**  $m\angle LJM = 35^\circ$



Statements	Reasons
1. $\overline{JL}$ bisects $\angle KJM$ .	1. Given
2. $\angle KJL \cong \angle LJM$	2. Definition of congruent angles
3. $m\angle KJL = m\angle LJM$	3. Definition of angle bisector
4. $m\angle KJL = 35^\circ$	4. Given
5. $m\angle LJM = 35^\circ$	5. Transitive Property

My Notes

a)

Statements	Reasons
$\angle 1$ and $\angle 2$ are vertical angles	Given
$\angle 1 \cong \angle 2$	Vertical angles are congruent
$\angle 1 \cong \angle 3$	Given
$\angle 2 \cong \angle 3$	Transitive Property

b)

Statements	Reasons
$\angle R$ and $\angle S$ are supplementary to $\angle T$	Given
$m\angle R + m\angle T = 180^\circ$ $m\angle S + m\angle T = 180^\circ$	Definition of Supplementary angles
$m\angle R + m\angle T = m\angle S + m\angle T$	Transitive Property
$m\angle R = m\angle S$	Subtraction Property of Equality
$\angle R \cong \angle S$	Definition of congruent angles

3) Reasons 2 and 3 do not make logical sense. The reasons are switched. The proof would be correct if he switched Reasons 2 and 3.

1)  $\angle D \cong \angle F$

**ACTIVITY 6**

continued

**Lesson 6-2**

**Two-Column Geometric Proofs**

My Notes

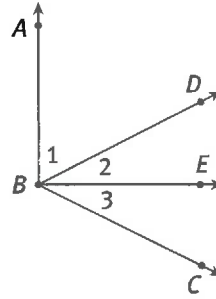
Statements	Reasons
M is the midpoint of $\overline{LN}$ .	Given
$\overline{LM} \cong \overline{MN}$	Definition of midpoint
$LM = MN$	Definition of congruent segments
$LM = 8$	Given
$MN = 8$	Transitive Property
$LN = LM + MN$	Segment Addition Postulate
$LN = 8 + 8 = 16$	Substitution

**LESSON 6-2 PRACTICE**

4. Supply the missing statements and reasons.

**Given:**  $\angle 1$  is complementary to  $\angle 2$ ;  $\overline{BE}$  bisects  $\angle DBC$ .

**Prove:**  $\angle 1$  is complementary to  $\angle 3$ .



Statements	Reasons
1. $\overline{BE}$ bisects $\angle DBC$	1. Given
2. $\angle 2 \cong \angle 3$	2. Definition of angle bisector
3. $m\angle 2 = m\angle 3$	3. Definition of congruent angles
4. $\angle 1$ is complementary to $\angle 2$ .	4. Given
5. $\angle 1 + m\angle 2 = 90^\circ$	5. Definition of complementary angles
6. $m\angle 1 + m\angle 3 = 90^\circ$	6. Substitution
7. $\angle 1$ is complementary to $\angle 3$ .	7. Definition of complementary angles

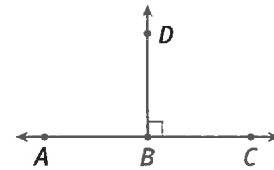
**Construct viable arguments.** Write a two-column proof for each of the following.

5. **Given:** M is the midpoint of  $\overline{LN}$ ;  $LM = 8$ .

**Prove:**  $LN = 16$

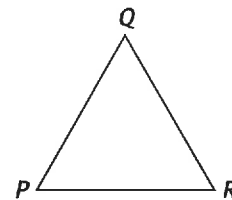
6. **Given:**  $\overline{BD}$  bisects  $\angle ABC$ ;  $m\angle DBC = 90^\circ$ .

**Prove:**  $\angle ABC$  is a straight angle.



7. **Given:**  $\overline{PQ} \cong \overline{QR}$ ,  $QR = 14$ ,  $PR = 14$

**Prove:**  $\overline{PQ} \cong \overline{PR}$



8. **Reason abstractly.** What type of triangle is shown in Item 7? Explain how you know.

**MATH TIP**

If the given information does not include a diagram, it may be helpful to make a sketch to represent the information.

8) Equilateral;  $\overline{QR}$  and  $\overline{PR}$  have the same length and  $\overline{PQ}$  is proven to be congruent. Therefore, all three sides have the same length.