Congruence Transformations and Triangle Congruence

Truss Your Judgment

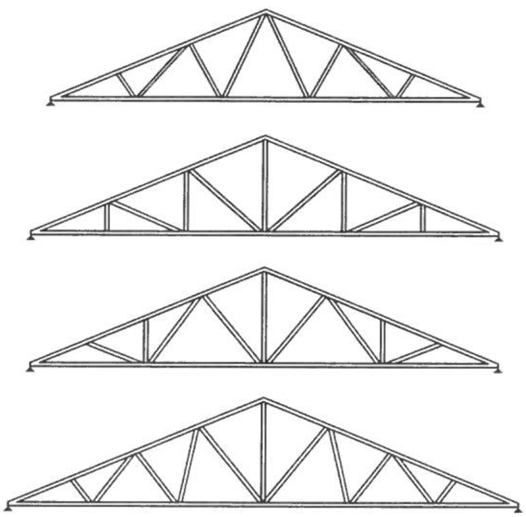
Lesson 11-1 Congruent Triangles

Learning Targets:

- Use the fact that congruent triangles have congruent corresponding parts.
- Determine unknown angle measures or side lengths in congruent triangles.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Debriefing, Use Manipulatives, Think-Pair-Share

Greg Carpenter works for the Greene Construction Company. The company is building a new recreation hall, and the roof of the hall will be supported by triangular trusses, like the ones shown below.



Each of the trusses contains pairs of congruent triangles. Greg's boss tells him that his first job will be to determine the side lengths and angle measures in the triangles that make up one of the trusses.

My Notes

CONNECT TO CAREERS

Triangles are often used in construction for roof and floor trusses because of their strength and rigidity. Each angle of a triangle is held solidly in place by its opposite side. That means the angles will not change when pressure is applied—unlike other shapes.

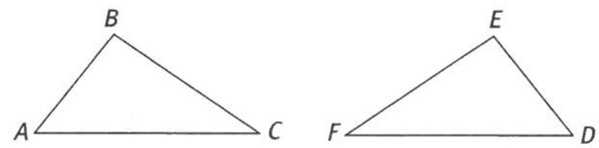
MATH TIP

Congruent triangles are triangles that have the same size and shape. More precisely, you have seen that two triangles are congruent if and only if one can be obtained from the other by a sequence of rigid motions.

My Notes

Greg wonders, "If I know that two triangles are congruent, and I know the side lengths and angle measures in one triangle, do I have to measure all the sides and angles in the other triangle?"

Greg begins by examining two triangles from a truss. According to the manufacturer, the two triangles are congruent.



1. Because the two triangles are congruent, can one triangle be mapped onto the other? If yes, what are the criteria for the mapping?

Each triangle can be mapped onto the other triangle by a sequence of rigid motions.

2. Suppose you use a sequence of rigid motions to map $\triangle ABC$ to $\triangle DEF$. Find the image of each of the following under this sequence of transformations.

$\overline{AB} \rightarrow$ $\overline{DE}$ $\quad$ $\overline{BC} \rightarrow$ $\overline{EF}$ $\quad$ $\overline{AC} \rightarrow$ $\overline{DF}$

$\angle A \rightarrow$ $\angle D$ $\quad$ $\angle B \rightarrow$ $\angle E$ $\quad$ $\angle C \rightarrow$ $\angle F$

3. Make use of structure. What is the relationship between $\overline{AB}$ and $\overline{DE}$? What is the relationship between $\angle B$ and $\angle E$? How do you know?

$\overline{AB} \cong \overline{DE}$ and $\angle B \cong \angle E$ because there is a sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$ and a sequence of rigid motions that maps $\angle B$ to $\angle E$.

MATH TERMS

Corresponding parts result from a one-to-one matching of sides and angles from one figure to another. Congruent triangles have three pairs of congruent sides and three pairs of congruent angles.

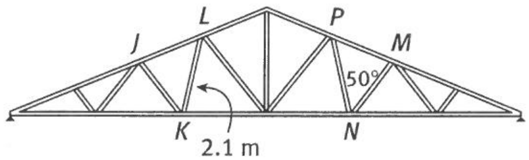
The triangles from the truss that Greg examined illustrate an important point about congruent triangles. In congruent triangles, corresponding pairs of sides are congruent and corresponding pairs of angles are congruent. These are ***corresponding parts***.

When you write a congruence statement like $\triangle ABC \cong \triangle DEF$, you write the vertices so that corresponding parts are in the same order. So, you can conclude from this statement that $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$, $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.

My Notes

Example A

For the truss shown below, Greg knows that $\triangle JKL \cong \triangle MNP$.



Greg wants to know if there are any additional lengths or angle measures that he can determine.

Since $\triangle JKL \cong \triangle MNP$, $\overline{KL} \cong \overline{NP}$. This means $KL = NP$, so $NP = 2.1$ m.

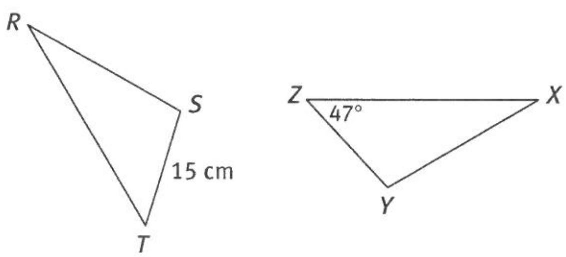
Also, since $\triangle JKL \cong \triangle MNP$, $\angle K \cong \angle N$. This means $m\angle K = m\angle N$, so $m\angle K = 50°$.

MATH TIP

Two line segments are congruent if and only if they have the same length. Two angles are congruent if and only if they have the same measure.

Try These A

In the figure, $\triangle RST \cong \triangle XYZ$. Find each of the following, if possible.



a. $m\angle X$
not possible

b. YZ 15 cm

c. $m\angle T$
47°

d. XZ
not possible

e. Both $\triangle JKL$ and $\triangle MNP$ are equilateral triangles in which the measure of each angle is 60°. Can you tell whether or not $\triangle JKL \cong \triangle MNP$? Explain.
No. Even though the angles are congruent, the sides may not be.

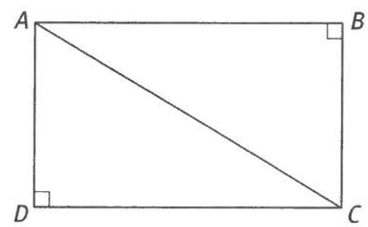
My Notes

4) Yes. Their side lengths are the same

5) No. Congruent triangles have the same angle measures.

Check Your Understanding

4. If two triangles are congruent, can you conclude that they have the same perimeter? Why or why not?
5. Is it possible to draw two congruent triangles so that one triangle is an acute triangle and one triangle is a right triangle? Why or why not?
6. Rectangle $ABCD$ is divided into two congruent right triangles by diagonal $\overline{AC}$.



Fill in the blanks to show the congruent sides and angles.

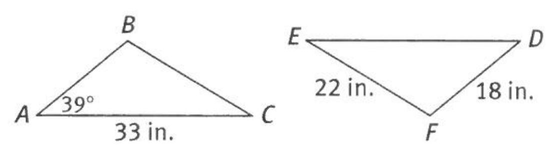
a. $\overline{AB} \cong$ $\overline{CD}$
b. $\overline{BC} \cong$ $\overline{DA}$
c. $\angle BAC \cong$ $\angle DCA$
d. $\angle ACB \cong$ $\angle CAD$

7. $\triangle PQR \cong \triangle GHJ$. Complete the following.

a. $\overline{QR} \cong$ $\overline{HJ}$
b. $\overline{GJ} \cong$ $\overline{PR}$
c. $\angle R \cong$ $\angle J$
d. $\angle G \cong$ $\angle P$

LESSON 11-1 PRACTICE

In the figure, $\triangle ABC \cong \triangle DFE$.



8. Find the length of $\overline{AB}$.
9. Find the measure of all angles in $\triangle DEF$ that it is possible to find.
10. What is the perimeter of $\triangle DEF$? Explain how you know.
11. **Construct viable arguments.** Suppose $\triangle XYZ \cong \triangle TUV$ and that $\overline{XY}$ is the longest side of $\triangle XYZ$. Is it possible to determine which side of $\triangle TUV$ is the longest? Explain.

8) 18 in.

9) $m\angle D = 39°$

10) 73 in.

11) Yes. Since $\overline{XY}$ and $\overline{TU}$ are corresponding sides, their side lengths must be congruent.

My Notes

Learning Targets:

- Develop criteria for proving triangle congruence.
- Determine which congruence criteria can be used to show that two triangles are congruent.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Use Manipulatives, Think-Pair-Share

As you have seen, congruent triangles have six pairs of congruent corresponding parts. The converse of this statement is also true. That is, if two triangles have three pairs of congruent corresponding sides and three pairs of congruent corresponding angles, the triangles are congruent.

Greg's boss asks him to check that two triangles in a truss are congruent. Greg wonders, "Must I measure and compare all six parts of both triangles?" He decides that a shortcut will allow him to conclude that two triangles are congruent without checking all six pairs of corresponding parts.

1. Greg begins by checking just one pair of corresponding parts of the two triangles.

a. In your group, each student should draw a triangle that has a side that is 2 inches long. The other two sides can be any measure. Draw the triangles on acetate or tracing paper.

b. To check whether two triangles are congruent, place the sheets of acetate on the desk. If the triangles are congruent, you can use a sequence of translations, reflections, and rotations to map one triangle onto the other.

Are all of the triangles congruent to each other? Why or why not?

The triangles are not congruent. No sequence of rigid motions will map one triangle to another.

c. Cite your results from part b to prove or disprove this statement: "If one part of a triangle is congruent to a corresponding part of another triangle, then the triangles must be congruent."

Students' triangles should serve as counterexamples to disprove the statement.

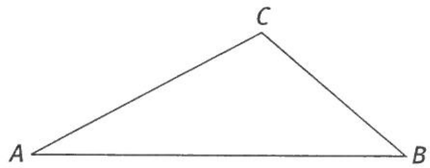
MATH TIP

A *counterexample* is a single example that shows that a statement is false.

My Notes

Now Greg wonders if checking two pairs of corresponding parts suffices to show that two triangles are congruent.

2. Greg starts by considering $\triangle ABC$ below.



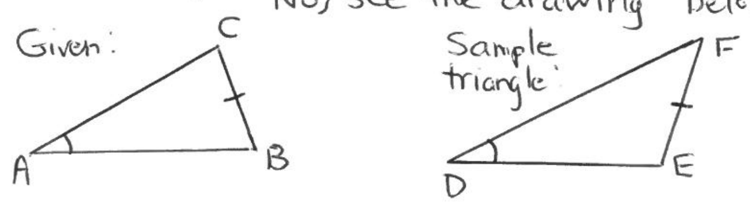
a. Draw triangles that each have one side congruent to $\overline{AB}$ and another side congruent to $\overline{AC}$. Use transformations to check whether every triangle is congruent to $\triangle ABC$. Explain your findings.

Every such triangle is not congruent to $\triangle ABC$. The angle that corresponds to $\angle A$ can vary between 0 and 180 degrees.

b. Draw triangles that each have an angle congruent to $\angle A$ and an adjacent side congruent to $\overline{AB}$. Is every such triangle congruent to $\triangle ABC$? Explain.

No. The side that corresponds to $\overline{AC}$ can have any length.

c. Draw triangles that each have an angle congruent to $\angle A$ and an opposite side congruent to $\overline{CB}$. Is every such triangle congruent to $\triangle ABC$? Explain. No; see the drawing below.



d. Draw triangles that each have an angle congruent to $\angle A$ and an angle congruent to $\angle B$. Is every such triangle congruent to $\triangle ABC$? Explain.

No. These triangles all have the same shape but may have different sizes.

e. Consider the statement: "If two parts of one triangle are congruent to the corresponding parts in a second triangle, then the triangles must be congruent." Prove or disprove this statement. Cite the triangles you constructed.

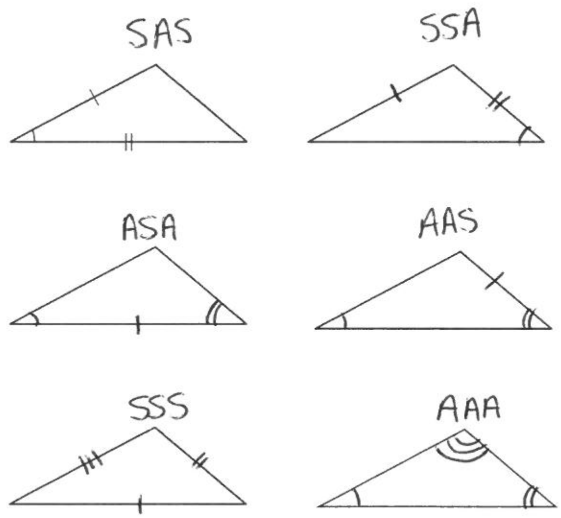
Disprove. Students' triangles serve as counterexamples.

My Notes

Greg decides that he must have at least three pairs of congruent parts in order to conclude that two triangles are congruent. In order to work more efficiently, he decides to make a list of all possible combinations of three congruent parts.

3. Greg uses *A* as an abbreviation to represent angles and *S* to represent sides. For example, if Greg writes *SAS*, it represents two sides and the included angle, as shown in the first triangle below. Here are the combinations in Greg's list: SAS, SSA, ASA, AAS, SSS, and AAA.

a. Mark each triangle below to illustrate the combinations in Greg's list.



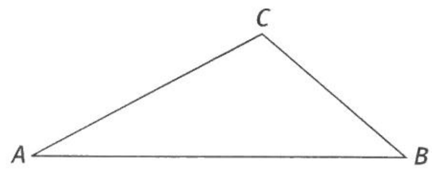
b. Are there any other combinations of three parts of a triangle? If so, is it necessary for Greg to add these to his list? Explain.

SAA and ASS; these are not necessary because SAA is the same as AAS and ASS and SSA are the same.

My Notes

Now Greg wants to find out *which* pairs of congruent corresponding parts guarantee congruent triangles.

4. Three segments congruent to the sides of $\triangle ABC$ and three angles congruent to the angles of $\triangle ABC$ are given in Figures 1–6, shown below.



a. If needed, use manipulatives supplied by your teacher to recreate the six figures given below.

Figure 1

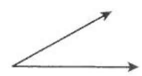


Figure 3

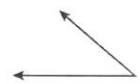


Figure 2

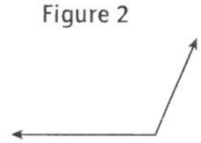


Figure 4 ______________________

Figure 5 ________________

Figure 6 ____________

b. Identify which of the figures in part a is congruent to each of the parts of $\triangle ABC$.

$\angle A$: Figure 1 $\angle B$: Figure 3

$\angle C$: Figure 2 $\overline{AB}$: Figure 4

$\overline{CB}$: Figure 6 $\overline{AC}$: Figure 5

My Notes

5. For each combination in Greg's list in Item 3, choose three appropriate triangle parts from Item 4. Each student should create a triangle using these parts. Then use transformations to check whether every such triangle is congruent to $\triangle ABC$. Use the table to organize your results.

Combination	Name the Three Figures Used by Listing the Figure Numbers	Is Every Such Triangle Congruent to $\triangle ABC$?
SSS	4, 5, 6	Yes
SAS	4, 1, 5 5, 2, 6 6, 3, 4	Yes
ASA	1, 5, 2 2, 6, 3 3, 4, 1	Yes
AAS	1, 2, 6 1, 3, 6 2, 1, 4 2, 3, 4 3, 1, 5 3, 2, 5	Yes
AAA	1, 2, 3	No
SSA	1, 5, 6 2, 6, 4 3, 4, 5 4, 5, 2 5, 6, 3 6, 4, 1	No

6. Express regularity in repeated reasoning. Compare your results from Item 5 with those of students in other groups. Then list the different combinations that seem to guarantee a triangle congruent to $\triangle ABC$. These combinations are called ***triangle congruence criteria***.

SSS
SAS
ASA
AAS

7. Do you think there is an AAA triangle congruence ***criterion***? Why or why not?

No. Two triangles with three congruent angles are not necessarily congruent. Information about at least one side length is needed.

ACADEMIC VOCABULARY

A ***criterion*** (plural: *criteria*) is a standard or rule on which a judgment can be based. Criteria exist in every subject area. For example, a scientist might use a set of criteria to determine whether a sample of water is safe for human consumption.

My Notes

Greg realizes that it is not necessary to check all six pairs of corresponding parts to determine if two triangles are congruent. The triangle congruence criteria can be used as "shortcuts" to show that two triangles are congruent.

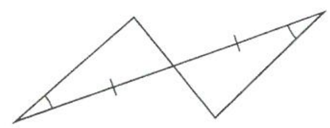
Example A

For each pair of triangles, write the triangle congruence criterion, if any, that can be used to show the triangles are congruent.

a.

Since three pairs of corresponding sides are congruent, the triangles are congruent by SSS.

b.



Although they are not marked as such, the vertical angles in the figure are congruent. The triangles are congruent by ASA.

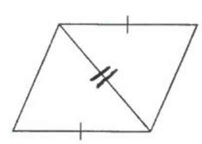
MATH TIP

You can use theorems about vertical angles, midpoints, bisectors, and parallel lines that are cut by a transversal to identify additional congruent parts that may not be marked in the figure.

Try These A

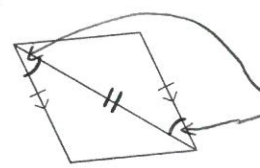
For each pair of triangles, write the triangle congruence criterion, if any, that can be used to show the triangles are congruent.

a.



none

b.



SAS

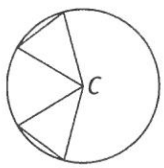
alternate interior angles are congruent because the lines are parallel

Check Your Understanding

8. Two triangles each have two sides that are 8 feet long and an included angle of 50°. Must the two triangles be congruent? Why or why not?

9. Two equilateral triangles each have a side that is 5 cm long. Is it possible to conclude whether or not the triangles are congruent? Explain.

10. The figure shows a circle and two triangles. For both triangles, two vertices are points on the circle and the other vertex is the center of the circle. What information would you need in order to prove that the triangles are congruent?



My Notes

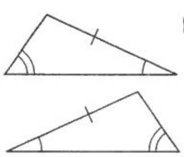
8) Yes. They are congruent by SAS.

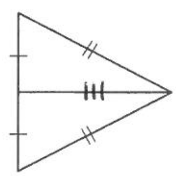
9) Yes They are congruent by SSS.

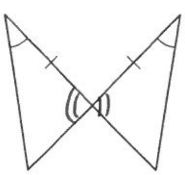
10) If the distance between the two vertices on the circle are the same for both triangles, then the triangles are congruent by SSS. If the angles at the vertex are congruent, then the triangles are congruent by SAS.

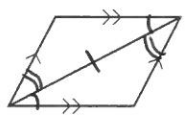
LESSON 11-2 PRACTICE

For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

11.  none

12.  SSS

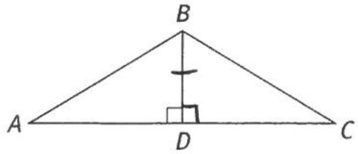
13.  ASA

14.  ASA

CONNECT TO AP

The free-response items on the AP Calculus exam will often ask you to *justify* your answer. Such justifications should follow the same rules of deductive reasoning as the proofs in this geometry course.

15. Make sense of problems. What one additional piece of information do you need, if any, in order to conclude that $\triangle ABD \cong \triangle CBD$? Is there more than one set of possible information? Explain.



15) If $\angle ABD \cong \angle CBD$, ASA
If $\angle A \cong \angle C$, AAS
If $\overline{AD} \cong \overline{CD}$, SAS
If $\overline{AB} \cong \overline{CB}$, HL

My Notes

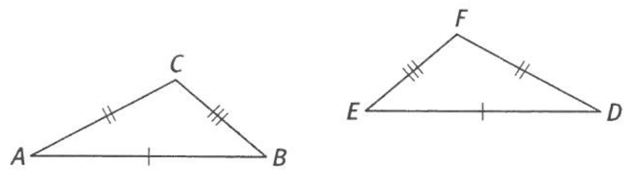
Learning Targets:

- Prove that congruence criteria follow from the definition of congruence.
- Use the congruence criteria in simple proofs.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Think Aloud, Discussion Groups, Visualization

You can use the SSS, SAS, or ASA congruence criteria as shortcuts to show that two triangles are congruent. In order to prove *why* these criteria work, you must show that they follow from the definition of congruence in terms of rigid motions.

1. To justify the SSS congruence criterion, consider the two triangles below.



a. What given information is marked in the figure?

$\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$

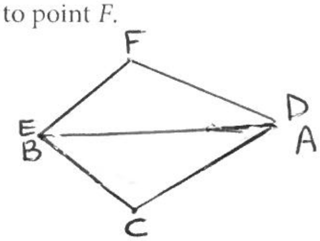
b. Based on the definition of congruence, what do you need to do in order to show that $\triangle ABC \cong \triangle DEF$?

Show that there is a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$.

c. It is given that $\overline{AB} \cong \overline{DE}$. What does this tell you?

There is a sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$.

d. Draw a figure to show the result of the sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$. Assume that this sequence of rigid motions does *not* map point C to point F.



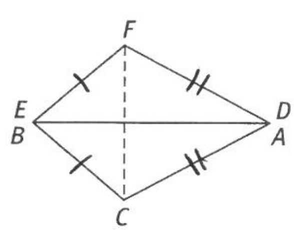
MATH TIP

You are asked to assume that the sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$ does not map C to F. If it did map C to F, you would have found a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$ and the proof would be complete!

e. Which points coincide in your drawing? Which segments coincide?

Points A and D; Points B and E; $\overline{AB}$ and $\overline{DE}$

f. Mark the line segments that you know are congruent in the figure below.



My Notes

g. Based on the figure, how is $\overline{ED}$ related to $\overline{FC}$? Why?

$\overline{ED}$ is the perpendicular bisector of $\overline{FC}$.

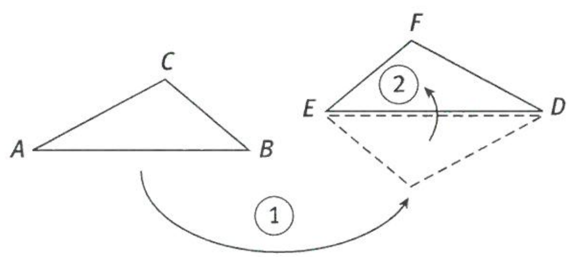
MATH TIP

The Perpendicular Bisector Theorem states that a point lies on the perpendicular bisector of a line segment if and only if it is equidistant from the endpoints of the segment.

h. Consider the reflection of $\triangle ABC$ across $\overline{ED}$. What is the image of $\triangle ABC$? How do you know?

The image of $\triangle ABC$ is $\triangle DEF$. Point C maps to point F since the line of reflection is the perpendicular bisector of $\overline{FC}$.

The argument in Item 1 shows that a sequence of rigid motions maps $\triangle ABC$ to $\triangle DEF$. Specifically, the sequence of rigid motions is a rotation that maps $\overline{AB}$ to $\overline{DE}$, followed by a reflection across $\overline{ED}$. By the definition of congruence, $\triangle ABC \cong \triangle DEF$.



2. In the proof, is it important to know exactly which rigid motions map $\overline{AB}$ to $\overline{DE}$? Explain.

Specifying the exact sequence of rigid motions is a useful strategy. It shows that such a rigid motion does exist.

3. Attend to precision. How is the definition of a reflection used in the proof?

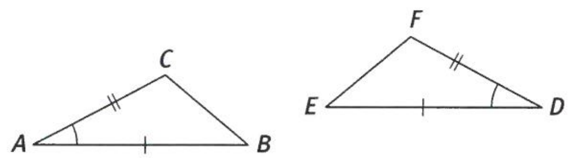
The line of reflection is used to show that $\overline{ED}$ is the perpendicular bisector of $\overline{FC}$.

My Notes

MATH TIP

Your proof should use all three pieces of given information that you identified in part a. If you find that you are not using all of these facts, you may be missing an element of the proof.

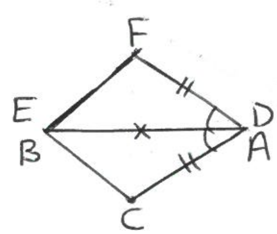
4. To justify the SAS congruence criterion, consider the two triangles below.



a. What given information is marked in the figure?

$\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\angle A \cong \angle D$

b. The proof begins in the same way as in Item 1. Since $\overline{AB} \cong \overline{DE}$, there is a sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$. Draw a figure at the right to show the result of this sequence of rigid motions. Assume that this sequence of rigid motions does *not* map point C to point F.



c. Mark the line segments and angles that you know are congruent in your figure.

d. Suppose you reflect $\triangle ABC$ across $\overline{ED}$. What is the image of $\overline{AC}$? Why?

The image of $\overline{AC}$ is $\overline{DF}$ because $\overline{ED}$ is the angle bisector of $\angle CDF$.

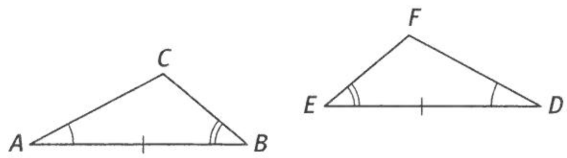
e. When you reflect $\triangle ABC$ across $\overline{ED}$, can you conclude that the image of point C is point F? Explain.

Yes, Point C must lie on $\overline{DF}$. The image of point C is equidistant from D and point F. The image of point C is point F.

The argument in Item 4 shows that there is a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Specifically, the sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$, followed by the reflection across $\overline{ED}$, maps $\triangle ABC$ to $\triangle DEF$. By the definition of congruence, $\triangle ABC \cong \triangle DEF$.

My Notes

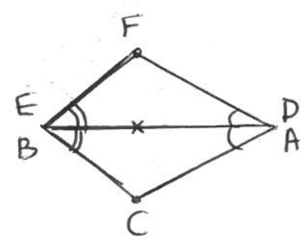
5. To justify the ASA congruence criterion, consider the two triangles below.



a. What given information is marked in the figure?

$\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, $\angle B \cong \angle E$

b. The proof begins in the same way as in Item 1. Since $\overline{AB} \cong \overline{DE}$, there is a sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$. Draw a figure at the left to show the result of this sequence of rigid motions. Assume that this sequence of rigid motions does *not* map point C to point F.



c. Mark the line segments and angles that you know are congruent in your figure.

d. Suppose you reflect $\triangle ABC$ across $\overline{ED}$. What is the image of $\overline{AC}$? What is the image of $\overline{BC}$? Why?

The image of $\overline{AC}$ is $\overline{DF}$ because $\overline{ED}$ is the angle bisector of $\angle CDF$. The image of $\overline{BC}$ is $\overline{EF}$ because $\overline{ED}$ is the angle bisector of $\angle CEF$.

e. When you reflect $\triangle ABC$ across $\overline{ED}$, can you conclude that the image of point C is point F? Explain.

Yes.

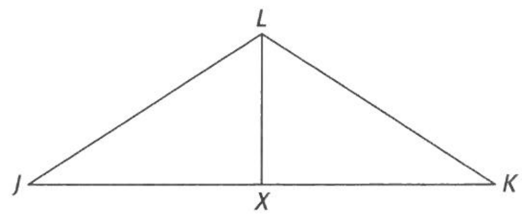
My Notes

The argument in Item 5 shows that there is a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Specifically, the sequence of rigid motions that maps $\overline{AB}$ to $\overline{DE}$, followed by the reflection across $\overline{ED}$, maps $\triangle ABC$ to $\triangle DEF$. By the definition of congruence, $\triangle ABC \cong \triangle DEF$.

Now you can use the SSS, SAS, and ASA congruence criteria to prove that triangles are congruent.

Example A

Greg knows that point X is the midpoint of $\overline{JK}$ in the truss shown below. He also makes measurements and finds that $\overline{JL} \cong \overline{KL}$. He must prove to his boss that $\triangle JXL$ is congruent to $\triangle KXL$.



Given: X is the midpoint of $\overline{JK}$; $\overline{JL} \cong \overline{KL}$.

Prove: $\triangle JXL \cong \triangle KXL$

Statements	Reasons
1. X is the midpoint of $\overline{JK}$.	1. Given
2. $\overline{JX} \cong \overline{KX}$	2. Definition of midpoint
3. $\overline{JL} \cong \overline{KL}$	3. Given
4. $\overline{LX} \cong \overline{LX}$	4. Congruence is reflexive.
5. $\triangle JXL \cong \triangle KXL$	5. SSS

Try These A

a. Suppose that Greg knew instead that $\overline{LX}$ was perpendicular to $\overline{JK}$ and suppose he made measurements to find that $\overline{JX} \cong \overline{KX}$. Could he prove to his boss that $\triangle JXL$ is congruent to $\triangle KXL$? If so, write the proof. If not, explain why not.

b. Suppose that Greg knew instead that $\overline{LX}$ bisects $\angle JLK$. Could he prove to his boss that $\triangle JXL$ is congruent to $\triangle KXL$? If so, write the proof. If not, explain why not.

a)

Statements	Reasons
1) $\overline{JX} \cong \overline{KX}$	Given
2) $\overline{LX} \perp \overline{JK}$	Given
3) $\angle LXK$ and $\angle LXJ$ are right angles.	Perp. lines intersect to form right angles.
4) $\angle LXK \cong \angle LXJ$	Right angles are congruent
5) $\overline{LX} \cong \overline{LX}$	Reflexive Property
6) $\triangle JXL \cong \triangle KXL$	SAS Congruence Postulate

6) Yes; by CPCTC

Check Your Understanding

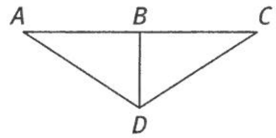
6. In the Example, can Greg conclude that $\angle J \cong \angle K$? Why or why not?
7. Draw a figure that contains two triangles. Provide given information that would allow you to prove that the triangles are congruent by the ASA congruence criterion.

LESSON 11-3 PRACTICE

For Items 8–10, write each proof as a two-column proof.

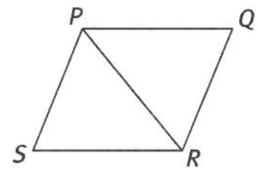
8. Given: $\overline{AD} \cong \overline{CD}$; $\angle ADB \cong \angle CDB$

Prove: $\triangle ADB \cong \triangle CDB$



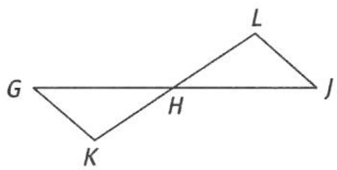
9. Given: $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$

Prove: $\triangle PQR \cong \triangle RSP$



10. Given: $\angle K \cong \angle L$; $\overline{KH} \cong \overline{LH}$

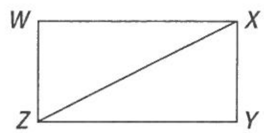
Prove: $\triangle GKH \cong \triangle JLH$



11. Critique the reasoning of others. A student wrote the proof shown below. Critique the student's work and correct any errors he or she may have made.

Given: $\overline{WX} \cong \overline{YZ}$; $\overline{ZW} \cong \overline{XY}$

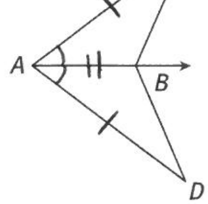
Prove: $\triangle ZWX \cong \triangle XYZ$



Statements	Reasons
1. $\overline{WX} \cong \overline{YZ}$	1. Given
2. $\overline{ZW} \cong \overline{XY}$	2. Given
3. $\angle W$ and $\angle Y$ are right angles.	3. Given
4. $\angle W \cong \angle Y$	4. All right angles are congruent.
5. $\triangle ZWX \cong \triangle XYZ$	5. SAS

12. Model with mathematics. A graphic designer made a logo for an airline, as shown below. The designer made sure that $\overline{AB}$ bisects $\angle CAD$ and that $\overline{AC} \cong \overline{AD}$. Can the designer prove that $\triangle ABC \cong \triangle ABD$? Why or why not?

Yes; by SAS Congruence Postulate



My Notes

8)

Statements	Reasons
1) $\overline{AD} \cong \overline{CD}$	Given
2) $\angle ADB \cong \angle CDB$	Given
3) $\overline{BD} \cong \overline{BD}$	Reflexive Property
4) $\triangle ADB \cong \triangle CDB$	SAS Congruence Postulate

9)

Statements	Reasons
1) $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$	Given
2) $\overline{PQ} \cong \overline{RS}$	Transitive Property
3) $\overline{QR} \cong \overline{SP}$	Transitive Property
4) $\overline{PR} \cong \overline{PR}$	Reflexive Property
5) $\triangle PQR \cong \triangle RSP$	SSS Congruence Postulate

10)

Statements	Reasons
1) $\angle K \cong \angle L$	Given
2) $\overline{KH} \cong \overline{LH}$	Given
3) $\angle LHJ \cong \angle GHJ$	Vertical Angles Congruence Theorem
4) $\triangle GKH \cong \triangle JLH$	ASA Congruence Postulate

My Notes

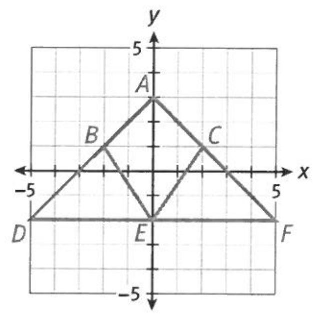
Learning Targets:

- Apply congruence criteria to figures on the coordinate plane.
- Prove the AAS criterion and develop the HL criterion.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Debriefing

You can use the triangle congruence criteria on the coordinate plane.

1. **Reason quantitatively.** Greg's boss hands him a piece of graph paper that shows the plans for a truss. Greg's boss asks him if he can prove that $\triangle DBE$ is congruent to $\triangle FCE$.



a. Use the distance formula to find each length.

$BD =$ $3\sqrt{2}$ $CF =$ $3\sqrt{2}$

$DE =$ 5 $FE =$ 5

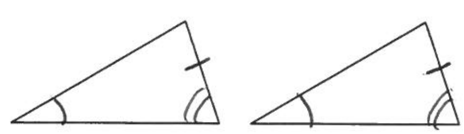
$BE =$ $\sqrt{13}$ $CE =$ $\sqrt{13}$

b. Can Greg use this information to prove that $\triangle DBE \cong \triangle FCE$? Explain.

Yes; $\triangle DBE \cong \triangle FCE$ by the SSS congruence criterion.

2. In Item 5 of Lesson 11-2, you discovered that SSS, SAS, and ASA are not the only criteria for proving two triangles are congruent. You also discovered that there is an AAS congruence criterion. What does the AAS congruence criterion state? Mark the triangles below to illustrate the statement.

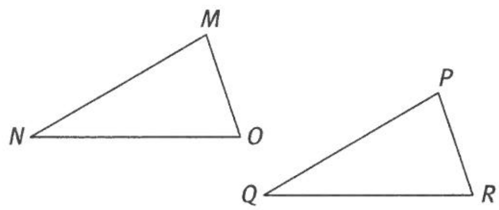
If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and nonincluded side of another triangle, then the triangles are congruent.



3. The proof of the AAS congruence criterion follows from the other congruence criteria. Complete the statements in the proof of the AAS congruence criterion below.

Given: $\triangle MNO$ and $\triangle PQR$ with $\angle N \cong \angle Q$, $\angle O \cong \angle R$, and $\overline{MO} \cong \overline{PR}$

Prove: $\triangle MNO \cong \triangle PQR$

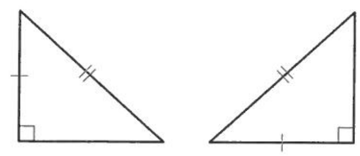


Statements	Reasons
1. $\triangle MNO$ and $\triangle PQR$	1. Given
2. $m\angle M + m\angle N + m\angle O = 180°$, $m\angle P + m\angle Q + m\angle R = 180°$	2. The sum of the measures of the angles of a triangle is 180°.
3. $m\angle M + m\angle N + m\angle O = m\angle P + m\angle Q + m\angle R$	3. Transitive Property of Equality
4. $\angle N \cong \angle Q$; $\angle O \cong \angle R$	4. Given
5. $m\angle N = m\angle Q$; $m\angle O = m\angle R$	5. Definition of congruent angles
6. $m\angle M + m\angle N + m\angle O = m\angle P + m\angle N + m\angle O$	6. Substitution Property of Equality
7. $m\angle M = m\angle P$	7. Subtraction Property of Equality
8. $\angle M \cong \angle P$	8. Definition of congruent angles
9. $\overline{MO} \cong \overline{PR}$	9. Given
10. $\triangle MNO \cong \triangle PQR$	10. ASA

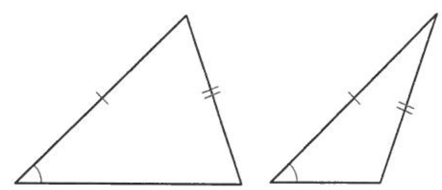
My Notes

4. Below are pairs of triangles in which congruent parts are marked. For each pair of triangles, name the angle and side combination that is marked and tell whether the triangles *appear* to be congruent.

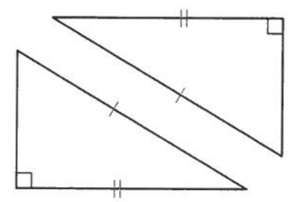
   a. SSA; yes



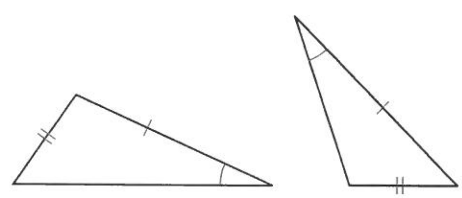
   b. SSA; no



   c. SSA; yes



   d. SSA; no

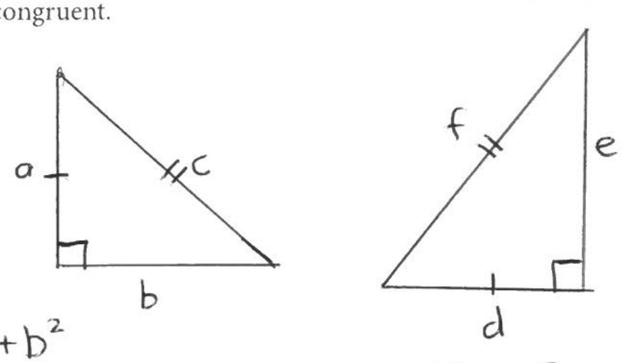


5. We know that in general SSA does not always determine congruence of triangles. However, for two of the cases in Item 4 the triangles appear to be congruent. What do the congruent pairs of triangles have in common?

   They are right triangles.

My Notes

6. In a right triangle, we refer to the correspondence SSA shown in Items 4a and 4c as *hypotenuse-leg* (HL). Write a convincing argument in the space below to prove that HL will ensure that right triangles are congruent.

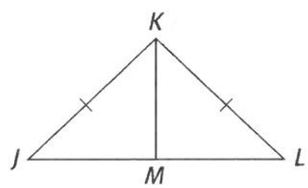


$c^2 = a^2 + b^2$
$f^2 = d^2 + e^2$
$c^2 = f^2$ (since $c = f$)
$a^2 + b^2 = d^2 + e^2$
$a^2 + b^2 = a^2 + e^2$ (since $a = d$)

Therefore, $b^2 = e^2$;
The triangles are congruent by SSS, SAS, or HL.

Check Your Understanding

7. Is it possible to prove that $\triangle LKM \cong \triangle JKM$ using the HL congruence criterion? If not, what additional information do you need?



8. Do you think there is a leg-leg congruence criterion for right triangles? If so, what does the criterion say? If not, why not? Review your answers. Be sure to check that you have described the situation with specific details, included the correct mathematical terms to support your reasoning, and that your sentences are complete and grammatically correct.

7) No. There is no proof that $\angle KMJ$ and $\angle KML$ are right angles.

8) It is not necessary because of HL.

My Notes

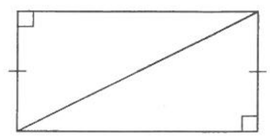
9) AB = $\sqrt{13}$ DE = $\sqrt{13}$
BC = $\sqrt{5}$ EF = $\sqrt{5}$
CA = $2\sqrt{5}$ FD = $2\sqrt{5}$
Congruent by SSS.

LESSON 11-4 PRACTICE

9. Construct viable arguments. On a coordinate plane, plot triangles ABC and DEF with vertices $A(-3, -1)$, $B(-1, 2)$, $C(1, 1)$, $D(3, -4)$, $E(1, -1)$, and $F(-1, -2)$. Then prove $\triangle ABC \cong \triangle DEF$.

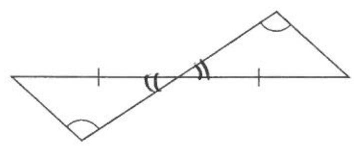
For each pair of triangles, write the congruence criterion, if any, that can be used to show the triangles are congruent.

10.



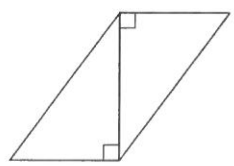
$\boxed{\text{HL}}$

11.



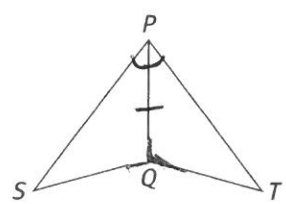
$\boxed{\text{AAS}}$

12.



$\boxed{\text{none}}$

13. $\overline{PQ}$ bisects $\angle SPT$.



$\boxed{\text{none}}$